Inflation Targeting under Fiscal Fragility

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Abstract

We propose a model to study an inflation-targeting regime under a high government debt burden. We assume that an altruistic policymaker chooses debt issuance, inflation, and public expenditure, while private agents dislike inflation and finance the government. We show that equilibrium inflation depends on debt level: (1) on-target when debt is low; (2) above the target when debt is high; (3) either above or on-target in between, a zone that we named fiscal fragility. Equilibrium inflation also depends on the target level: a higher target may improve welfare by preventing fiscal fragility and reducing debt-rollover costs.

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1 Introduction

The inflation targeting regime became the cornerstone of central bank coordination of inflation expectations after its widespread adoption in the 1980s and 1990s. However, in both advanced and emerging economies, episodes of coordination failures in which inflation expectations suddenly diverge from the announced targets are common.[1] Some of these episodes lack sizable changes in fundamentals that would explain the shift in expectations, raising questions on the limits of inflation targeting to anchor short-term inflation expectations. Since the seminal contribution of Calvo (1988), several authors proposed explanations for how a sudden deterioration in inflation expectations can disanchor inflation from a lower level.[2] None of these proposed a dynamic model considering an explicit role of the inflation target level choice in avoiding a self-fulfilling inflationary crisis.

We propose a dynamic model that rationalizes these episodes of coordination failures and self-fulfilling inflation occurring in indebted economies under an inflation targeting regime. In our model, the inflation target level is central to determining the policymaker's ability to anchor expectations and avoid loss of confidence in the monetary regime. A lower inflation target reduces inflation welfare costs but increases the debt-rollover cost required for target accomplishment. Therefore, lower inflation targets may increase the temptation of the policymaker to deviate to a higher level of inflation to alleviate fiscal constraints, which opens doors to a confidence crisis. A higher inflation target, on the other hand, raises inflation welfare costs but also makes agents believe in the policymaker's ability to deliver the target by reducing the policymaker's temptation to abandon the monetary regime.

Our model consists of a closed economy in which two types of agents, an altruistic policymaker and private agents, act rationally in an environment with complete information. The policymaker acts jointly as a fiscal authority and central bank, pursuing an inflation target by choosing current inflation and financing government expenditures by taxing and selling debt. We assume that the policymaker is not perfectly committed to the inflation target and might deviate from it to make fiscal room for spending. Its decision is the solution to the tradeoff between inflating public debt away and keeping inflation on target to avoid the economic costs of deviating. Private agents choose how much debt to hold and form expectations about next-period inflation. Our framework builds on Calvo (1988), Araujo, Berriel, and Santos (2016), and Cole and Kehoe (1996, 2000), expanding their analysis to an inflation targeting setting.

[1] Roger and Stone (2005) note that targets are often missed (40% in their sample) and sometimes "by substantial amounts and for prolonged periods." Based on our updated data set, used in Appendix B, we conclude that targets are still frequently missed (26%) in a sample including recent years.

[2] See Araujo and Leon (2002), Araujo, Leon, and Santos (2013), Corsetti and Dedola (2016), and Bianchi and Mondragon (2021) for examples of models in which a self-fulfilling crisis can occur in a monetary setting with sovereign debt economies.

The economy's fiscal side and policymaker's discretion are fundamental to understanding the capacity of the inflation-targeting regime to coordinate inflation expectations. We show that the actual inflation decision depends on endogenous debt zones. When public debt is sufficiently low, the interest burden is low, and government spending is high. Therefore, private agents believe in the target, and the policymaker ensures it. On the other extreme, when debt is sufficiently high, the benefit of abandoning the inflation target exceeds the cost of maintaining it. For an intermediary endogenous interval of the debt level, the target may or may not be delivered since the policymaker is subject to confidence crises. We named this zone the fiscal fragility zone (FFZ). Within the FFZ, multiple equilibria exist, leading to an expected inflation rate that exceeds the inflation target. This results in higher debt service costs and reduced government spending.

A policymaker under fiscal fragility is subject to high debt service costs to deliver the pre-announced target. In our model, for a given inflation target, the ex-ante rollover cost always equals the inverse of the intertemporal discount factor, which we denote as $1/\beta$. This arises because we assume that risk-neutral agents price government bonds rationally. However, in the fiscal fragility zone, the policymaker might deviate from the pre-announced target, which results in an inflation expectation above the target. Consequently, the ex-post rollover cost can either be higher or lower than $1/\beta$. It is higher when the target is delivered since actual inflation is lower than expected inflation, but it can also be lower when the inflation overshoots the target and partial default happens.

In the fiscal fragility zone, a higher inflation target reduces the rollover cost when the policymaker delivers the target. In particular, this finding stems from incentives in place when the policymaker deviates from the target. A higher target reduces the marginal benefit of inflating the debt since, after a partial default, the extra government revenue gained from target deviation depends on a ratio between the discretionary inflation and the inflation target. Therefore, for the same amount of discretionary inflation level, the extra revenue that the government obtains by partially defaulting on the debt is decreasing at the target level. We show this result formally in Proposition 4.[3]

Private agents internalize that a higher inflation target reduces debt costs and that maintaining the target is now less costly for the policymaker. As a result, we show that the floor of the fiscal fragility zone increases with the target level, allowing the target to be assured in higher debt levels. We prove this results formally in Proposition 5 under the assumption of increasing and nonconcave inflation welfare costs. Therefore, higher inflation targets can

[3] In the Appendix, we test our model predictions using a panel dataset of 20 countries with at least 15 years of inflation targeting. We find evidence that the size of deviations from the target and the probability of deviating are negatively related to the target level. We also find evidence that deviations from the target are positively related to the debt level.

prevent confidence crises.

Fiscal policy also has an important role during inflation coordination crises in our model. Depending on the debt levels, the optimal policy may involve either gradually reducing public debt to eliminate confidence crises and support the inflation-targeting regime or gradually increasing public debt and awaiting the confidence crises. Also, debt composition has a role. We show that, when allowing for real debt in our model, the policymaker can avoid self-fulling inflation by issuing a sufficiently high fraction of real debt. However, issuing high levels of real debt may lead the policymaker not to honor its obligations, a situation considered in detail by the sovereign debt literature (Eaton and Gersovitz, 1981; Cole and Kehoe, 2000; Arellano, 2008).

We calibrate our model based on Brazil's response to inflationary pressures at the end of 2002. Figure I illustrates this episode with the expected inflation minus the inflation target on the y-axis and when expected inflation was formed on the x-axis. The three lines show expected inflation across three horizons relevant to monetary policy: the end of the current year, one year ahead, and two years ahead. Until October 2002, expected inflation was within the inflation target bands. After that, inflation expectations exceeded the target's upper bounds at all horizons, indicating a target confidence crisis. The crisis was triggered when it became clear that the candidate favorite to win the election could come with a new policy framework. In response to rising inflation expectations, policymakers reacted in accordance with the recommendations that align with the ones of our model. In particular, the inflation target for 2003 was increased twice, and a more sustainable fiscal policy was established.

Insert Figure I about here.

Our paper provides practical implications for the design of the monetary policy framework. Based on the calibrated model, we show that the optimal inflation target is the lowest level possible for the economy to immediately exit the fiscal fragility zone. Importantly, the optimal inflation target depends on the debt level since more indebted economies need higher inflation target levels to exit the FFZ. In light of this result, it seems naïve to choose a 2% inflation target without considering fiscal fundamentals as countries eventually do.

Our model explains why some indebted economies set higher targets to escape the fiscal fragility zone, and others often miss these targets because they are prone to using inflation to finance public expenditures during crises. Recently, many developed economies missed their inflation target after increasing debt levels in response to the recent pandemic crisis. For example, Barro and Bianchi (2023) study a sample of OECD countries and empirically conclude that approximately 40-50% of the financing effort for the increases in public expenditure during the pandemic years came from inflation surprise. Their results are consistent with our model predictions.

Related Literature

Our model contributes to the literature on the ability of the inflation-targeting regime to anchor inflation expectations and provide a stable inflation level for indebted economies. In the conventional view, synthesized in the seminal work of Woodford (2003), monetary policy can stabilize inflation around a predetermined target path as long as the monetary authority commits to a specific policy trajectory. The debate over the optimal inflation target focuses on determining the level of inflation that maximizes economic welfare. This target ranges from a negative rate, as suggested by the Friedman rule, to a positive rate, as suggested by models considering the zero lower bound and sticky prices. See Schmitt-Grohé and Uribe (2010), Coibion, Gorodnichenko, and Wieland (2012), and Adam and Weber (2019), for examples. This framework is adequate to study the problem of setting a target in economies on a sustainable fiscal path but might not be ideal for countries with a high debt burden in which inflation serves as a partial default mechanism. We add to this literature by showing that the inflation target level might affect monetary policy by changing the benefits and costs of abandoning the inflation targeting regime and that a higher target might increase welfare.

Our message that the inflation target level is important to understand inflation expectations coordination in indebted economies also adds to the extensive literature on the fiscal limits of monetary policy and on the interdependence between fiscal discipline and price stability. Sargent and Wallace (1981) highlight the importance of the fiscal side in understanding inflation control, followed by contributions from Leeper (1991), Sims (1994, 2011), Woodford (1995), Leeper and Leith (2016), Araujo, Berriel, and Santos (2016), Cochrane (2018) and Bianchi, Faccini, and Melosi (2023). From those, Araujo, Berriel, and Santos (2016) are the closest to our paper and also have the result that a lower inflation target results in high costs of maintaining the monetary regime. We differ from them in that we have a dynamic model with complete information and specified preferences for the private agents, while they consider a static model with imperfect information and an *ad-hoc* loss function. The simplicity of their model limits its usefulness for empirically checking their theoretical statements.

This literature on fiscal-monetary interdependence has shown that fiscal rules play an important role in determining a unique equilibrium in dynamic rational expectations models and, in particular, New-Keynesian (NK) models. For example, Cochrane (2011, 2022) shows that, in NK models, the Taylor principle does not solve the inflation indeterminacy, and explosive inflation paths are still equilibria. He argues in favor of a fiscal theory of price level, in which fiscal policy has an active role and determines the price level through the valuation equation of government debt, and the central bank has a passive role and follows an interest rate rule that does not destabilize the economy. Others assume escape-clause rules to eliminate this multiplicity of equilibria: a different policy regime to which the government switches if inflation exits certain bounds.

We contribute to this literature by offering a different perspective, focusing on an altruistic policymaker who strategically decides inflation and fiscal policy. In our model, there is no conflict between fiscal and monetary authorities since both monetary and fiscal policies act to maximize private agents' welfare. The policymaker's lack of commitment to pursuing the inflation target creates an interaction between the policymaker's optimal policy and private agents' expectations, which potentially leads to multiple equilibria. In this regard, our multiple equilibria differ from those that the fiscal theory of price level or escape clause rules address. Our model with no conflict between fiscal and monetary authorities also differs from the model of Bianchi (2019) and Bianchi and Melosi (2022), who model an economy with a fiscal-monetary conflict arising from inconsistencies between fiscal and monetary objectives

We also contribute to this literature by incorporating into a simple DSGE a policymaker who decides monetary policy strategically, does not follow a Taylor rule, and may use inflation to partially default. This approach closely follows papers on confidence crises in debt markets (Cole and Kehoe, 1996, 2000; Calvo, 1988). Other papers have explored debt crises and their relation to monetary policy (Uribe, 2006; Aguiar, Amador, Farhi, and Gopinath, 2013; Corsetti and Dedola, 2016; Bacchetta, Perazzi, and van Wincoop, 2018; Arellano, Mihalache, and Bai, 2019). No paper in this literature has considered the relationship between fiscal fundamentals and the inflation target level.

Similar to Cole and Kehoe (1996, 2000), our model has a multiple equilibria zone that depends on the debt level. We differ from their model by explicitly modeling a monetary regime where the government borrows from domestic private agents using domestic-currency bonds. Crises happen when the government has high debt and fails to meet its inflation target, thereby partially defaulting. Our model also differs from most of the quantitative sovereign debt models that followed Arellano (2008) and Aguiar and Gopinath (2006). Our baseline model assumes the government can only issue domestic debt and can partially default on it by choosing an inflation level above the pre-determined inflation target. The equilibrium depends on the realization of a sunspot variable when the economy is within the fiscal fragility zone.

Next Sections

In Section 2, we set out the model and define the equilibrium. We define the discretionary inflation chosen by the policymaker when deviating from the target and prove that it is increasing in the debt level. We then obtain endogenous debt zones that characterize the behavior of the policymaker and prove that the optimal policy is stationary outside of the fiscal fragility zone, where multiple equilibria are possible. We finish the section by proving that the real interest rate decreases in the inflation target when the economy is in the FFZ and that the lower bound of the FFZ increases in the target level.

In Section 3, we specify functional forms and parameter values in a quantitative analysis to match the Brazillian situation in 2002. We obtain the optimal inflation target for each debt level. In the numerical exercises, the optimal target is the lowest possible level at which the economy exits the FFZ. In Section 4, we analyze the 2002 confidence crisis in Brazil and the subsequent policy responses through the lens of our model. It shows that the actions taken by the government to anchor inflation expectations are consistent with the results predicted by our model. In Section 5, we consider an extension of our model where a fraction of the debt is real. Finally, the last section presents concluding remarks.

2 Model

We consider a closed economy with two types of agents: a policymaker and private agents. Each agent lives infinite periods and forms rational expectations with complete information. We assume that the policymaker is altruistic and maximizes private agents' welfare. It acts as a combined fiscal and monetary authority, choosing current inflation and selling one-period debt to finance itself. In our setup, the inflation choice is reduced to a discrete choice each period of whether to deliver a pre-announced inflation target or deviate from it. Private agents choose how much debt to hold and form expectations about next-period inflation while considering the inflation target and the current debt level. In each period, they receive a stream of fixed endowments. When multiple equilibria are possible, a sunspot variable determines the equilibrium.

2.1 Basic Setup

Policymaker

We consider an altruistic policymaker who chooses both fiscal and monetary policies to maximize private agents' utility. As a monetary authority, the policymaker chooses the inflation rate π_t, which generates disutility. It targets an exogenously set inflation target π^a, assumed to be selected during the adoption of the inflation targeting regime. As a fiscal authority, the policymaker chooses the next period's debt D_{t+1}. The policymaker's problem is:

$$\max_{\pi_t, D_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t, g_t) - \psi(\pi_t) \right] \;, \tag{1}$$

where c_t is private agents' consumption in period t, g_t is government spending on public goods, and β is the intertemporal discount rate, $0 < \beta < 1$. Consumption and public goods are nonnegative. We define private agents' utility as a weighted average of linear consumption and government spending utility similar to Cole and Kehoe (2000). The weights are defined by the parameter $\rho \in (0, 1)$ that can be interpreted as a relative preference for private consumption:

$$u(c_t, g_t) = \rho c_t + (1 - \rho) v(g_t) \;,$$

where v is a twice-differentiable strictly increasing and strictly concave function of g satisfying $\lim_{g \to 0^+} v(g) = -\infty$.

Linearity in consumption results in an equilibrium real interest rate that depends only on the inflation level and target. As will be explained further, the policymaker makes decisions strategically considering the impact of its actions on the equilibrium real interest rate. The linearity assumption implies that the real rate is independent of private consumption levels, which, otherwise, would be an additional variable for the policymaker to track in its inflation and debt decisions. This simplification makes the problem significantly more tractable both analytically and computationally. Linearity in consumption also simplifies the problem by making the debt stationary outside the crisis zone (to be defined) and readily makes the marginal utility of public goods higher than the marginal utility of consumption since public spending is constrained by debt interest spending and the fixed tax rate.

The disutility of inflation ψ captures the welfare and output costs of higher inflation levels and is independent of the remaining variables of the model, except, possibly, of the inflation target π^a. We assume that ψ is separable from the utility function u for mathematical convenience, as we can obtain similar results under more general assumptions, so long as we maintain the previous assumption of linearity in private consumption. Due to the linear consumption utility specification, we interpret the disutility of inflation as a cost in terms of consumption goods of higher inflation due to, for example, price adjustment costs.[4] It is assumed to satisfy $\psi' > 0$ and $\psi'' \geq 0$. Convexity of the disutility function is a necessary assumption to guarantee an interior solution for the discretionary inflation chosen by the

[4]See Bailey (1956) and Lucas (2000) for examples of models that find a way for inflation levels to affect welfare and output. Cysne (2009) shows that Bailey's measure provides a measure of the welfare costs of inflation derived from an intertemporal general-equilibrium model, while Campos and Cysne (2018) estimate output costs of inflation for the Brazilian case.

policymaker.[5] When the disutility function depends on the target, we also assume that $\psi'' + \frac{1+\pi^a}{1+\pi}\frac{\partial \psi'}{\partial \pi^a} \geq 0$, which implies that the marginal disutility of inflation is non-decreasing on the inflation level when delivering the target. This hypothesis accommodates a wide array of possible disutility functions that may depend on the inflation level, on the target level, or on the deviation from the target, such as $\psi(\pi) = \kappa(\pi - \pi^a)^2$, for a constant parameter κ. In the numerical exercises, we specify a simple quadratic cost $\psi(\pi) = \kappa\pi^2$, for simplicity, without compromising general results.

In each period, the policymaker finances the nonnegative spending g_t and the repayments on previous-period obligations through a fixed tax rate τ on a deterministic endowment e and the issuance of new debt D_{t+1}.[6] [7] First, we assume an initial level of debt that can be thought of as inherited from a previous government, which may have used fiscal policy to respond to a past recession or address public calamities such as wars or the COVID-19 pandemic. Second, we assume that the tax level is not high enough so that the level of public spending is below the one that equates to marginal spending between public and private consumption. In other words, the marginal utility of spending in public consumption is larger than that of spending in private consumption, which is equal to ρ given our linear utility in consumption. We can write this hypothesis mathematically as $(1-\rho)v'(\tau e) \geq \rho$. The government's budget constraint is given by

$$g_t + (1+r_t)D_t \leq D_{t+1} + \tau(e - \epsilon_t)\ , \tag{2}$$

where D_t is the last-period debt, and r_t is the real interest rate.[8] [9]

The fixed endowment is subject to a penalty ϵ_t that depends on the policymaker's choice of inflation. ϵ_t is a permanent fixed cost that affects the economy if the policymaker chooses

[5]In the Online Appendix C, we also consider a variation of the model in which inflation decreases the endowment level instead of being a utility cost, with similar results.

[6]The fixed tax rate hypothesis can be interpreted as a situation in which the policymaker has no additional space to increase taxes to reduce indebtedness without significantly affecting output. This situation is similar to what is observed in a middle-income economy with relatively high tax levels such as Brazil. Alternatively, this hypothesis can be interpreted as an impossibility of adjusting taxes during a crisis.

[7]Our emphasis in the paper is on the debt rollover cost. As a result, the proper concept of debt (gross or net) may depend on the context of the country analyzed. For example, in the presence of foreign reserves accumulation, the debt rollover cost will depend on the spread of the domestic interest rate over foreign reserves return. For emerging economies, such as the Brazilian economy, the return on foreign reserves is small compared to the high interest rates on government debt, so the gross debt level is a better indicator of indebtedness in the economy. This may not be the case in advanced economies with a lower spread. For simplification, we refrain from explicitly modeling the reserves accumulation decision.

[8]We restrict our analysis to initial debt levels that leave the policymaker with a nonempty set of feasible choices, $D_t \in [0, D^{max}]$, where D^{max} is high enough. For very high initial debt levels, the policymaker could have no way of satisfying the positive constraint on c and g. To see this, suppose that debt servicing costs are higher than tax revenues, $\left(\frac{1}{\beta} - 1\right) D_0 > \tau e$, which leaves no space for spending. Then, even if the policymaker were to partially default on debt payments, it would still be unable to meet future positive spending restrictions due to the high future debt servicing costs and the inability to use inflationary surprises again.

[9]We also assume a no-Ponzi condition, so the government cannot run-up infinite debt in the long run: $\lim_t \beta^t D_{t+1} = 0$.

to deviate from the target.[10] This fixed cost is of the form

$$\epsilon_t = \begin{cases} 0 & \text{if } \pi_t = \pi^a \text{ and } \epsilon_{t-1} = 0, \\ \epsilon & \text{otherwise.} \end{cases}$$

We consider ϵ to be a high but bounded penalty cost that is always higher than the inflation cost when inflation is on the target. This implies that $\epsilon > \psi(\pi^a)/\rho$ or that the penalty for deviating is always higher than the inflation disutility of delivering the target (in terms of private consumption). We also assume that ϵ is sufficiently high to avoid results of target deviations from a relaxed penalty assumption and bounded to avoid unreal perfect commitment.

In each period, the policymaker can satisfy the budget constraint by i) adjusting expenditures, ii) issuing new debt D_{t+1}, and iii) partially defaulting on debt through an inflationary surprise, π_t, and rolling over the remaining debt. Given a nominal interest i_t defined at t, an inflationary surprise reduces the ex-post real interest rate and, consequently, the payments the policymaker makes on its debt. Such a partial default offers additional fiscal room for government spending.

To see how inflation decreases the real interest rate, observe that the ex-post real interest rate is given by:

$$1 + r_t = \frac{1 + i_t}{1 + \pi_t}\,, \tag{3}$$

where i_t is the nominal risk-free interest rate on bonds issued in period $t-1$. Increases in π_t decrease r_t given i_t.

An additional comment is due regarding the inflation target π^a. In the model, the target is exogenously set at the moment of the adoption of the inflation-targeting regime, with private agents not anticipating any changes to it. We later conduct both comparative statics analysis and quantitative sensitivity analyses regarding different levels of the target, which clarify the role of the inflation target in our framework. These assumptions align well with how the inflation target regime works in reality. Typically, the monetary committee establishes inflation targets for two to three years in advance, which the central bank then pursues. The fact that, in our model, the inflation target is taken as given is consistent with how this regime operates. Moreover, once set, altering the target is not easy, requiring an extraordinary committee meeting and, in some jurisdictions, even a parliamentary hearing. Therefore, the assumption that agents do not internalize the possibility of changing the target in response

[10] Our approach of assuming an exogenous functional form for the "cost of violating" the inflation target is in line with the literature. Exogenous penalty functions are also assumed in self-fulfilling debt crisis models as in Cole and Kehoe (1996, 2000) and in sovereign default models as in Arellano (2008). A positive violation cost gives an incentive to the policymaker to follow the inflation target. We interpret the penalty function as a reduced and parsimonious form of capturing the negative impacts of inflation deviation on economic activity.

to an unexpected change in the macroeconomic scenario, such as a major confidence crisis, is also consistent with how this regime operates

Private Agents

We assume a continuum of infinitely lived private agents who choose consumption and savings to maximize their expected utility:

$$\max_{c_t, d_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \ , \tag{4}$$

Since private agents do not choose the inflation level, the disutility function ψ is omitted from their maximization problem.

Each period, private agents receive a deterministic endowment e, subject to a cost of violating the inflation target ϵ_t, and payments on their bond holdings. The endowment is taxed at a constant rate τ by the government. The private agents' budget constraint is given by:

$$c_t + d_{t+1} \leq (1 + r_t)d_t + (1 - \tau)(e - \epsilon_t) \tag{5}$$

where d_{t+1} represents one-period bonds bought in t and d_t is the previous-period bond holdings paying the interest rate $(1 + r_t)$. The first-order condition for debt demand from the private agents problem implies the following equilibrium nominal interest-rate:

$$1 + i_t = \frac{1}{\beta} \frac{1}{\mathbb{E}_{t-1}\left[\frac{1}{1+\pi_t}\right]} \tag{6}$$

Private agents also form their expectations of the inverse gross inflation rate, $q_t^e = \mathbb{E}_{t-1} q_t$ where $q_t = (1 + \pi_t)^{-1}$. Since the policymaker has a binary choice of either following the previously announced target π^a or deviating to a discretionary inflation level π^D, the inflation expectation will be equal either to one of these levels or a probability-weighted sum of them, depending on the sunspot variable to be introduced. As a result, the expectation q_t^e will have a one-for-one correspondence to the expected inflation level $\pi_t^e = \mathbb{E}_{t-1}\pi_t$. The expectations formed will depend on the timing of actions.

Timing

Rational expectations govern the strategic interactions between the policymaker and private agents. As in Calvo (1988) and Cole and Kehoe (1996, 2000), multiple self-fulfilling equilibria may occur where, conditional on the debt level, the best response from the policymaker's perspective depends on the expectations of private agents. If private agents expect a deviation from the target, the best response will be to deviate. If they expect no deviations, the best response will be to keep inflation on target. In the multiple-equilibria case, we consider an exogenous sunspot variable ζ_t to determine the selection of the equilibrium. The sunspot variable determines which of the possible inflation rates will be the actual inflation rate π_t implemented by the government when there are two equilibrium rates: the inflation target π^a and discretionary inflation π_t^D.

At the beginning of each period, uncertainty is resolved through the realization of the sunspot variable ζ_t. The policymaker, considering the sunspot variable previously drawn, chooses how much debt D_{t+1} to sell and the inflation rate π_t, which will either be the target π^a or the discretionary inflation rate π_t^D. Finally, private agents form their expectations about the next period's inflation, π_{t+1}^e, which determines the expected inverse gross inflation rate q_{t+1}^e, and consequently nominal interest rate i_{t+1}, and decide the level of debt d_{t+1}. In summary, the timing of the model is:

1^{st} The sunspot variable ζ_t is realized.

2^{nd} The policymaker chooses actual inflation π_t, given sunspot ζ_t.

3^{rd} The policymaker chooses the next debt level D_{t+1}.

4^{th} Private agents form next-period inflation expectations π_{t+1}^e and choose the amount of next-period debt d_{t+1} to hold.

Given this timing, private agents may face uncertainty over which equilibrium will be selected in the next period when forming their inflation expectations. They will form expectations over each outcome's probability, considering the sunspot variable's exogenous distribution that determines the actual inflation rate. Inflation expectations will therefore be $\pi_t^e = f\pi_t^D + (1-f)\pi^a$, which imply an inverse gross inflation expectation of $q_t^e = f\frac{1}{1+\pi_t^D} + (1-f)\frac{1}{1+\pi^a}$, where f is the exogenously determined probability of the policymaker deciding to deviate from the inflation target due to an adverse situation. We interpret this negative sunspot as a change in the political landscape or a foreign event that deteriorates confidence in the monetary regime. Agents do not believe that the policymaker will deliver the target anymore.

Discretionary Inflation

We motivate the existence of deviations from the inflation target by modeling an altruistic policymaker who might choose an inflation level higher than the inflation target as a way of transferring resources for increasing public spending.[11] In each period, the policymaker may choose to deviate from the inflation target π^a, and private agents understand this when forming their expectations π_t^e. We call the inflation rate chosen by the policymaker when deviating from the target *discretionary inflation*. It is the result of a tradeoff between increasing government spending via higher inflation against the cost of facing a higher disutility of inflation. Let π_T^D be the endogenous and optimal level of discretionary inflation chosen at the time T of the deviation.

We assume that once the policymaker deviates from the inflation target, private agents lose confidence in the commitment of the policymaker to the target. Consequently, after the policymaker deviates, the economy enters a steady state because there is no longer any uncertainty to be resolved. The optimal fiscal policy is to maintain constant debt, such as $D_t = D_{T+1} \, \forall \, t > T$, as shown below in Proposition 2. Finally, the inflation disutility function takes the value $\psi(\pi^D)$ when deviating and remains so thereafter, while the cost of violating the inflation target is fixed at $\epsilon_t = \epsilon$ forever. The problem the policymaker resolves when defining the level of discretionary inflation can be written as follows:

$$\max_{\pi, D} u(c_T, g_T) + \frac{\beta}{1-\beta} u(c, g) - \frac{1}{1-\beta} \psi(\pi)$$

subject to

$$\begin{aligned} g_T &= \tau(e - \epsilon) - (1 + r_T^D) D_T + D \\ g &= \tau(e - \epsilon) - D \left(\frac{1}{\beta} - 1 \right) \\ c_T &= e - \epsilon - g_T \\ c &= e - \epsilon - g \\ 1 + r_T^D &= \frac{1}{\beta} \frac{1}{q_T^e} \frac{1}{1+\pi}. \end{aligned} \tag{7}$$

Where $1 + r_T^D$ is the ex-post real interest rate when the policymaker chooses to deviate. The optimal discretionary inflation level π_T^D is the solution to the problem above given an initial debt level D_T.[12] Given rational expectations, in equilibrium, π_T^D is optimal given π_T^e and vice

[11] We do not model mechanisms of partial default on local currency domestic debt other than inflation, although governments have opted for alternatives such as reduction of principal or lower coupons (Reinhart and Rogoff, 2008).

[12] To avoid unnecessary notation, we drop the time subscript T on inflation and the debt level whenever there is no ambiguity.

versa.[13]

The necessary first-order condition for D plus the hypothesis of concavity of u with respect to g readily implies that the policymaker will set a stationary debt level D such that the stationary public consumption equals public consumption at time T, that is, $g_T = g$. Consequently, we have that the optimal stationary debt is equal to

$$D = \beta(1 + r_T^D)D_T \tag{8}$$

that is, inflationary surprises ($\pi > \pi_T^e$) reduce steady-state debt, decreasing the debt burden and allowing for higher public consumption both in the short and long-run, which implies a steady-state level of public spending of

$$g = \tau(e - \epsilon) - (1 - \beta)(1 + r_T^D)D_T. \tag{9}$$

We can rewrite the above equation as

$$(1 + r_T^D)D_T = \frac{1}{1-\beta}\left(\tau(e - \epsilon) - g\right), \tag{10}$$

which corresponds to the intertemporal budget constraint of the government: the left-hand side is the real value of government debt due in period T, which must be equal to the present value of future government surpluses. Choosing the inflation level π pins down the stationary spending level g that satisfies equation (10).

To gain intuition for the optimal choice of π, consider now the first-order condition of the policymaker's problem for choosing discretionary inflation. The first-order condition, already substituting for the steady-state level of debt, is

$$(\rho - (1-\rho)v'(g))\frac{\partial(1 + r_T^D)}{\partial \pi}D_T - \frac{1}{1-\beta}\psi'(\pi) = 0 \tag{11}$$

The first term represents the net marginal benefit of allocating spending to public goods through the inflationary surprise, which is positive since by assumption $(1-\rho)v'(g) \geq \rho$ for all feasible g, and the real interest rate is decreasing on the inflation level. The second term is the total intertemporal disutility cost of higher inflation on the economy. The discretionary

[13] We numerically solve this problem by writing it as a fixed point. First, we assume an initial $\pi_{T,0}^e = \pi^a$ and then find the optimal $\pi_{T,1}^D$. We update $\pi_{T,1}^e$ using $\pi_{T,1}^D$ according to the inflation expectation formation process of the private agents. If $\pi_{T,1}^e \neq \pi_{T,0}^e$, the problem is iterated to find the new optimal $\pi_{T,2}^D$ given $\pi_{T,1}^e$. We continue this process until $|\pi_{T,i-1}^e - \pi_{T,i}^e| < \eta$, where η is a small number. The existence of a rational expectation of inflation π^e given the optimal discretionary inflation π^D chosen by the policymaker is shown in Online Appendix D.

inflation chosen by the policymaker will balance out the marginal benefit of an increase in the public goods consumption that can be purchased through an increase in inflation with the marginal cost of a permanent higher inflation disutility.

Proposition 1 ***(Discretionary Inflation is Increasing in Debt)****: If π_T^D is an interior solution to the problem (7), the discretionary inflation level π_T^D is increasing in the initial debt level D_T:* $\frac{\partial \pi_T^D}{\partial D_T} > 0$.

Proof: see Appendix A.1.

Higher debt levels increase interest spending, reducing available funds for public consumption, and as a result increasing marginal utility of an additional public consumption unit. It also increases the fiscal space opened by deviating, raising the first term in the left-hand side in (11) for any given inflation level, while the second term, the marginal inflation disutility, is unchanged. The resulting discretionary inflation level that satisfies the first-order condition is higher, reflecting a higher marginal benefit of deviating from the target.

2.2 Recursive Equilibrium

We define a recursive equilibrium where the policymaker and private agents sequentially choose their actions. At the beginning of each period, the aggregate state $s = (D, \pi^e, \zeta, \epsilon_{-1})$ is public since the aggregate debt D, the expected inflation for the current period π^e, and the past penalty ϵ_{-1} have all been determined in the previous period, while the realization of the sunspot variable ζ is determined in the start of the period before the agents decide.[14] The policy choices, π and D', the expected inflation for the next period $\pi^{e\prime}$, and the individual debt holdings for the next period d' determine the equilibrium jointly with s. We denote by $\pi(\cdot)$ and $D(\cdot)$ the inflation and debt policy functions, by $r(\cdot)$ the real interest rate function, and by $\pi^e(\cdot)$ the inflation expectation function, all yet to be defined.

To define a recursive equilibrium, we work backward on the timing of actions in each period. We start the definition of a recursive equilibrium with private agents because they move last. When forming expectations $\pi^{e\prime}$ at the end of any period, private agents know all their public debt holding d, the aggregate state s, the policymaker's offer of new debt D', current-period inflation π, and the policymaker's optimal policy functions. The following

[14] We let $\zeta = 1$ denote the occurrence of the negative sunspot, which is described below, and $\zeta = 0$ otherwise.

functional equation defines a private agent's value function:

$$
\begin{aligned}
V^{pa}(s, d, \pi, D') = \max_{c, d'} u(c, g) + \beta \mathbb{E} V^{pa}(s', d', \pi', D'') \\
\text{subject to} \\
& c + d' \leq (1 + r(s, \pi))d + (1 - \tau)(e - \epsilon(\pi, \pi^a, \epsilon_{-1})) \\
& s' = \Big(D', \pi^e(s, d, \pi, D'), \epsilon(\pi, \pi^a, \epsilon_{-1}), \zeta'\Big) \\
& \pi' = \pi(s') \\
& D'' = D(s') \\
& c \geq 0 \\
& d' \geq 0
\end{aligned}
\tag{12}
$$

in which we assume that private agents cannot sell public debt. The penalty function $\epsilon(\cdot)$ is a function of its previous value ϵ_{-1}, the inflation target π^a, and current inflation π.

Each period after the policymaker decides how much debt D' to offer and the inflation rate π, private agents decide how much debt to hold. Let $d'(s, d, \pi, D')$ be their debt policy function. When forming inflation expectations, private agents determine the nominal interest rate for the next period. In the absence of multiple equilibria, they perfectly anticipate π, and the real return is always $1/\beta$. If multiple equilibria are possible, private agents do not know what the policymaker will opt to do.

When forming inflation expectations, private agents consider what the policymaker could do in the next period. Their expectations are defined as $\pi^e(s, d, \pi, D') = \mathbb{E}\, \pi(s')$, where the expectation is conditional on all information available to the agent at the moment. When forming expectations, the set $(D', \pi^{e\prime}, \epsilon) \in s'$ is known to private agents. Hence, the only unknown variable on which private agents form their expectations is the realization of the sunspot variable ζ'. Integrating out the sunspot variable's commonly known distribution, we have

$$
\mathbb{E}\, \pi(s') = \begin{cases} f \times \pi^D(D', \pi^{e\prime}, \epsilon) + (1 - f) \times \pi^a & \text{if multiple eq.} \\ \pi^D(D', \pi^{e\prime}, \epsilon) & \text{if deviating unique eq.} \\ \pi^a & \text{if not deviating unique eq.} \end{cases} \tag{13}
$$

where f is the exogenous probability of the adverse equilibrium occurring and $\pi^D(D', \pi^{e\prime}, \epsilon)$ is the discretionary inflation chosen by the government when deviating given $(D', \pi^{e\prime}, \epsilon) \in s'$.

The policymaker chooses, at the beginning of the period, inflation π and debt issuance D', given state s. The policymaker knows that the next period's debt level affects the private agents' inflation expectations and resolves the following problem:

$$V^p(s) = \max_{\pi, D'} u\Big(c(s,d,\pi,D'), g\Big) - \psi(\pi) + \beta \mathbb{E} V^p(s')$$

subject to

$$\begin{aligned} & g + (1 + r(s,\pi))D \leq D' + \tau(e - \epsilon(\pi, \pi^a, \epsilon_{-1})) \\ & s' = (D', \pi^e(s,d,\pi,D'), \epsilon(\pi, \pi^a, \epsilon_{-1}), \zeta') \\ & g \geq 0 \end{aligned} \tag{14}$$

We can now define a recursive equilibrium for our model economy. An equilibrium is a list of value functions for the representative private agent V^{pa} and for the policymaker V^p; functions $c(\cdot)$ and $d'(\cdot)$ for the private agents' consumption and saving decisions; functions $\pi(\cdot)$ and $D'(\cdot)$ for the policymaker's inflation and debt decisions; an inflation expectation function $\pi^e(\cdot)$; a real interest rate function $r(\cdot)$; and an equation of motion for the aggregate debt level D' such that the following holds:

- Given D' and π, V^{pa} is the value function for the solution to the representative private agents' problem with c, d' and $\pi^{e\prime}$ being the maximizing choices when $d' = D'$.
- Given q^e, V^p is the value function for the solution to the policymaker problem, and both D' and π are the maximizing choices.
- $D'(s)$ equals $d'(s,d,\pi,D')$.

Our definition of an equilibrium is similar to that of Cole and Kehoe (1996) and Cole and Kehoe (2000) and is restricted to a Markov equilibrium. Future conditional plans of the agents can be derived from their policy functions.

2.3 The Fiscal Fragility Zone

The ability of the policymaker to effectively target inflation is restricted by debt levels. Assuming that inflation has always been on target, three different scenarios can be drawn according to the debt level D:[15]

- The *no crisis zone* $[0, \underline{\mathrm{D}}]$: D such that $V^p(D, \pi_t = \pi^a, \pi^e = \pi^D) \geq V^p(D, \pi_t = \pi^D, \pi^e = \pi^D) \rightarrow \pi_t = \pi^a = \pi^e$.

[15] With slight abuse of notation, we denote by $V^p(D, \pi, \pi^e)$ the total intertemporal utility attained by the policymaker by choosing inflation level π, given debt level D and private agents' expected inflation π^e, assuming that $\epsilon_{-1} = 0$ and that the sunspot $\zeta = 0$.

- The *fiscal fragility zone* $[\underline{D}, \overline{D}]$: D such that $\pi \in \{\pi^a, \pi^D\}$ depends on the sunspot.

- The *fiscal dominance zone* $[\overline{D}, D^{max}]$: D such that $V^p(D, \pi_t = \pi^D, \pi^e = \pi^a) \geq V^p(D, \pi_t = \pi^a, \pi^e = \pi^a) \rightarrow \pi_t = \pi^D = \pi^e$.

In the first case, the policymaker finds it preferable to keep inflation on target even when private agents believe that it will not. Consequently, only one equilibrium is possible where private agents have faith in the policymaker delivering on the target inflation. Since there is only one optimal choice for the policymaker regardless of the private agents' expectations, the sunspot ζ is disregarded, and the only important variable defining the policymaker's value function V^p is the debt level D. The same holds for the third case when the only equilibrium is the policymaker always deviating from the inflation target.

Whenever the policymaker is in the no-crisis zone or the fiscal dominance zone, it will always choose a stationary debt policy, as shown in Proposition 2:

Proposition 2 ***(Stationary Policy Outside of the Fiscal Fragility Zone):*** *The optimal debt policy chosen by the policymaker outside of the FFZ at period S is stationary, that is, $D_t = D_S$ for all $t \geq S$.*

Proof: see Appendix A.2.

When the policymaker deviates, private agents lose confidence forever in the ability of the government to maintain inflation targeting and, therefore, expect discretionary inflation, $\pi^e = \pi^D$. It is as if the economy enters the fiscal dominance zone. The above proposition justifies why we are allowed to consider only a stationary debt policy in the definition of discretionary inflation (7), as the fiscal dominance zone is identical to the steady state once the policymaker deviates from the target. On the other hand, in the no crisis zone, agents expect that the policymaker will follow the target, and it is optimal for the government to do so. In this case, due to the strict concavity of the utility function with respect to government spending, it is optimal for the policymaker to maintain debt level and spending constant, as agents dislike fluctuating government spending.

The more interesting scenario is multiple equilibria akin to self-fulfilling target failures. If private agents believe that the target will be delivered, then the policymaker will prefer to do so. On the contrary, in the face of adverse expectations, the policymaker chooses to deviate. In this zone, private agents have doubts about the commitment of the monetary authority to the target. The equilibrium is chosen by the realization of a sunspot, something the government binds its choice to but is unrelated to any observable fundamentals, and can be interpreted as a change in the political landscape or a foreign event that deteriorates confidence in the policymaker delivering the target.

When government debt is within the FFZ, the discretionary inflation rate is higher than the inflation target, as shown in Proposition 3:

Proposition 3 ***(Deviations from the Target are Positive):*** *In the Fiscal Fragility Zone the optimal deviation $\pi^D - \pi^a$ is always positive.*

Proof: see Appendix A.3.

If agents believe there is a probability that the government will deviate to a discretionary inflation level $\pi^D > \pi^a$, then expected inflation π^e is higher than the target. Then, the policymaker faces a higher cost of debt service and lower government spending to maintain the inflation target. To see this, let us recall the real interest rate on bonds from Equation (3). The real interest rate in the FFZ when the policymaker delivers the target will be given by:

$$1 + r_t^a = \frac{1}{\beta} \frac{1}{f \frac{1+\pi^a}{1+\pi^D} + 1 - f} \tag{15}$$

which is higher than the interest rate outside this zone, $1/\beta$. Thus, the government has an incentive to raise spending in the FFZ, which can only be done through inflation or increasing debt.

The cost of violating the inflation target ϵ plays a critical role in determining the existence of the debt zones and, ultimately, the inflation choice by the policymaker. If it is too low, there is no incentive for the policymaker to deliver the target, and the optimal choice will be to deviate from the discretionary inflation π^D regardless of debt level. In this case, all debt levels will be in the fiscal dominance zone. On the other hand, if the cost is too high, the welfare of deviating will be too low, as both private and government consumption will be significantly reduced. As a result, the policymaker will have no incentive to deviate, and agents will anticipate this outcome and set their expectations to the target so that all debt levels will be in the no-crisis zone. Therefore, a moderate level of violation cost will generate a discontinuity on the value function as a function of debt, such that for moderate debt levels, there will be multiple equilibria: the fiscal fragility zone.

The Inflation Target Coordination Role

A policymaker facing a higher inflation target has a lower ability to raise spending through deviating from the target, while it faces a lower cost of delivering the target, in the FFZ, as shown in Proposition 4:

Proposition 4 ***(Debt Rollover Cost is Decreasing in the Target Level when Delivering the Target):*** *Let $1 + r_t^a$ denote the real interest rate in the FFZ when the policymaker delivers the inflation target, and $1 + r_t^D$ when it deviates to the discretionary inflation. Then, $\frac{\partial(1+r_t^a)}{\partial \pi^a} < 0$ and $\frac{\partial(1+r_t^D)}{\partial \pi^a} > 0$.*

Proof: see Appendix A.4.

The intuition for this result is as follows: conditional on a moderate violation cost ϵ, a higher target induces the policymaker to raise π_T^D to attain the same level of reduction in interest expenditure for a given level of debt. Private agents anticipate the higher target and discretionary inflation and demand a higher (nominal) interest rate to purchase government bonds. Recall that the optimal stationary debt choice when deviating is given by

$$D = \frac{1}{f + (1-f)\frac{1+\pi^D}{1+\pi^a}} D_T. \tag{16}$$

Given a higher target π^a, the reduction in real debt provided by an additional inflation rate is diminished, since it is proportional to $1/(1+\pi^a)$. This implies a lower marginal benefit of raising inflation. But since $\psi'' \geq 0$, the marginal cost of an additional inflation rate is nondecreasing. A decreasing marginal benefit combined with a nondecreasing marginal cost results in a lower discretionary inflation. As a result, expected inflation is closer to the target, so that real rates raise when the policymaker deviates and fall when it delivers the target.

As a consequence, a higher inflation target can coordinate better inflation expectations toward the target rate. Faced with a lower benefit of deviating from the target, the policymaker will be less susceptible to inflating the public debt. Private agents rationalize this outcome and set their inflation expectations for the target, which makes delivering the target more accessible for the policymaker. The economy may exit the FFZ if the initial debt level is not too high. This is summarized in the following Proposition:

Proposition 5 ***(Fiscal Fragility Zone Floor raises with Higher Target):*** *The lower limit of the FFZ $\underline{D}$ is increasing in the inflation target level π^a.*

Proof: see Appendix A.5.

Propositions 4 and 5 depend crucially on the fact that

$$\frac{\partial \frac{1+\pi^D}{1+\pi^a}}{\partial \pi^a} < 0.$$

By taking the natural logarithm on the ratio $1+\pi^D/1+\pi^a$ and using the fact that $\log(1+x) \approx x$ for small x, we see that, up to an approximation, this result is equivalent to a decreasing deviation $\pi^D - \pi^a$ as the target is raised.

3 Quantitative Analysis

In this section, we calibrate the model based on the 2002 confidence crisis in Brazil. The presidential election of 2002 is an interesting case study in that the candidate most likely to win was running on a platform that appeared likely to deteriorate the fiscal situation. Professional forecasters surveyed by the central bank predicted inflation exceeding the target for all horizons. This loss of confidence in the government's ability to deliver the inflation target in the face of a perceived fiscally fragile situation is the type of event our model is designed to capture.

3.1 Functional Forms and Calibration

We assume that the government spending utility function is $v(g) = \log(g)$, and that inflation disutility function is given by $\psi(\pi) = \kappa\pi^2$, where κ is a constant. We consider a disutility function independent of the target to obtain an ex-ante optimal inflation target for each initial debt level.

Our model is calibrated on a yearly frequency to match the usual time frame targeted by central banks, and almost all parameters correspond to observable values during the 2002 confidence crisis in Brazil. First, we set the inflation target, π^a, to 3.5%, the official target that prevailed in 2002. Second, the discount factor, β, is 0.914 to match the historical average of the ex-post real interest rate between 1996 and 2019.[16] Third, the tax rate on endowments, τ, equals the 2002 general government revenue over GDP, 0.35. Fourth, we set the endowments, e, to 1.5 so that the public spending marginal utility of the total tax revenues is not lower than the private consumption marginal utility: $(1-\rho)v'(\tau e) \geq \rho$. Finally, we choose a neutral value for consumption preference $\rho = 1/2$ for the baseline exercises. We will use this parameter to obtain some static comparative results later.

[16] We compute the ex-post real interest rate using the interest rate on 1-year swaps, adjusted by the consumer price index. The average rate was 9.4% from 1996 to 2019, which we use to calibrate our model.

To construct this rate, we utilize data on the yield of 1-year nominal swaps provided by B3, the main stock exchange in Brazil. We adjust this yield by the annual IPCA (Índice Nacional de Preços ao Consumidor Amplo), the consumer price index employed by the Central Bank of Brazil for its inflation targeting regime. Our analysis spans from 1996 to 2019 to ensure that our calibration reflects a structural characteristic of the Brazilian economy rather than selective periods or specific economic conditions. However, computing the real interest rate using inflation-indexed bonds around 2002 yields a similar average interest rate (9.9%).

The exogenous crisis probability, f, is set at 20%, as Chan-Lau (2006) estimated for Brazil's sovereign risk during the election period. This risk is extracted from USD-denominated sovereign bonds. This default risk is a broad measure that encompasses inflationary default risk, but not exclusively. It happens to be that the frequency with which Brazil failed to meet its inflation target is also close to 20% (6 times in 25 years).

The parameter κ of the inflation disutility function is set according to Campos and Cysne (2018)'s estimation of a 0.35% of GDP cost for inflation of 10% in recent Brazilian experience, which we adjust to disutility by calculating the utility loss. The fixed cost of deviating ϵ equals 0.004, meaning a permanent 0.27% of GDP penalty for deviating from the target, and it is set to jointly match the gross debt level for the FFZ and the inflation index observed in Brazil in 2002. The crisis zone starts at approximately 70% of the debt ratio, slightly below the debt observed in 2002. Table I summarizes the chosen values.

Insert Table I about here.

3.2 Results

An indebted and altruistic policymaker optimally choosing inflation may deviate from the target in the event of an expectation shock. In our calibrated model, the policymaker becomes subject to such shocks after reaching a debt-to-GDP ratio of 70%. Below 70% of the debt ratio, the policymaker always prefers to keep inflation on target. For debt levels exceeding this lower bound, the equilibrium depends on the private agents' expectations, and the policymaker may decide to deviate given a negative sunspot shock. Taking this probability into account, private agents will demand higher nominal interest rates on government bonds once the policymaker exceeds this lower bound debt level. Finally, for debt levels exceeding 95% of GDP, the policymaker will always deviate from the target.

Optimal Fiscal Policy

The policymaker's optimal debt path depends upon the initial value of its debt stock. Outside the FFZ, it prefers to maintain debt levels constant, as shown in Proposition 2. Within the FFZ, it might i) choose fiscal responsibility and run down its debt to avoid the costs of an adverse equilibrium; ii) maintain constant debt levels; or iii) increase its debt to maintain a given spending level. In Figure II, we plot the next period's debt as a function of current debt. The three possible responses of the policymaker are seen within the FFZ. Those results are similar to those of Cole and Kehoe (1996).

Insert Figure II about here.

For a moderate initial debt level within the FFZ, the policymaker chooses a fiscally responsible debt path to avoid the expected endowment loss from deviating from the inflation target in the eventuality of a negative sunspot. In this region, expected inflation is higher than the target rate, which means that the policymaker faces a higher real interest rate than in the no crisis zone. However, as long as a negative sunspot shock that removes the confidence in the policymaker does not hit the economy, the optimal fiscal policy is to gradually reduce the debt-to-GDP ratio until the economy exits the FFZ. As the policymaker follows this austerity policy, expected inflation gradually declines, reducing the real interest rate burden and making it easier for the economy to exit the FFZ. This can be noted by observing that the slope of the policy function decreases as debt-to-GDP approaches the lower bound of the FFZ. Table II presents the expected inflation rates and the corresponding number of periods required to exit the FFZ for different levels of initial debt:

Insert Table II about here.

For a high initial debt level and fixed inflation target, the policymaker gradually reduces debt to return to the no-crisis zone. However, it takes a significant number of periods for the policymaker to regain private agents's confidence, during which it faces expected inflation rates higher than the target. The optimal policy for stabilizing inflation expectations in an environment of high indebtedness results in higher inflation expectations for a significant amount of time. Gradual exiting as an optimal decision is also present in Cole and Kehoe (1996, 2000) and in Sims (2020). The latter explicitly makes this prescription when claiming that it is not wise to reduce debt quickly by contracting fiscal expenditure. This result is in line with many episodes in which countries that experienced sudden increases in their debt-to-GDP ratios needed to stabilize their economies through higher temporary inflationary rates. Hall and Sargent (2022) describe the US post-war experience and attribute an important role in reducing the real value of debt to increases in price levels. Barro and Bianchi (2023) find that, for a sample of OECD countries, approximately 40-50% increases in public expenditure during the pandemic years were financed by inflation surprise.

Nevertheless, as the debt level grows, the fiscal room available to the policymaker shrinks due to the increased interest burden. Eventually, it is more desirable to run up debt to maintain spending. This situation happens above debt levels of 89%, as seen in Figure II. An austerity policy to exit the FFZ is not optimal, and the policymaker eventually suffers an adverse shock and loses private agents' confidence in the regime. By opting to run up debt, the policymaker will ultimately fail to give the needed fiscal support to the inflation target.

Coordinating Expectations Through the Target

Higher inflation targets may improve confidence in the monetary policy and help coordinate private agents' expectations by increasing the costs of deviating to attain a given inflationary transfer of resources. Private agents use the inflation target to form expectations in the FFZ, and the target functions as a nominal anchor for expectations. In this subsection, we present several sensitivity analyses of what happens with our model when we change the exogenously set inflation target. In these analyses, we assume that agents do not take the possibility of this change into account when forming expectations. Because we are interested in changing the target as a response to the confidence crises, which are rare events with high uncertainty, we believe it is reasonable to assume that agents do not expect it.

In Figure III, we present the sensitivity analysis of the deviations $\pi^D - \pi^a$ to changes in the inflation target. We plot these deviations for three different inflation targets (2%, 3.5%, and 5%), keeping the other parameters at their baseline.

Insert Figure III about here.

A higher inflation target improves coordination by the policymaker by reducing the discretionary deviation from the target rate and by reducing the real fiscal burden of debt through a lower real interest rate. A higher target rate reduces the marginal capacity of the policymaker to transfer resources, which implies a lower marginal benefit from discretionary inflation. As a consequence, a policymaker with a higher inflation target faces a lower real interest rate on its bonds in the FFZ, as shown in Proposition 4, and chooses a smaller deviation from the target.

For baseline parameters, deviations $\pi^D - \pi^a$ decrease in the inflation target, reducing the ex-post real interest rate in the FFZ. For initial debt levels below the lower bound of the FFZ, $\underline{\mathrm{D}}$, the policymaker will have a perfectly believable target, preferring to keep inflation on target regardless of private agent expectations. Above the lower bound, private agents may doubt its commitment. As deviations decrease in the target, keeping inflation on target for a given debt level becomes less costly. This effect increases the confidence in the inflation target because it remains fully assured up to higher debt levels, as shown in Proposition 5. In Figure IV, we plot the next-period debt for different inflation targets.

Insert Figure IV about here.

The lower bound $\underline{\mathrm{D}}$ increases as the target rate rises. This result implies that policymakers should consider current debt levels and fiscal conditions when deciding to decrease the inflation target. This reduction can cause a loss of confidence in the government's commitment as it enters the FFZ. Within the FFZ, expected inflation is higher than the target inflation rate, and choosing a low inflation target is costly instead of optimal. This result is in line with Araujo, Berriel, and Santos (2016), where a lower inflation target might reduce the policymaker's coordination ability due to a loss of confidence in its commitment and result in a worse equilibrium outcome.

The above analysis suggests a tradeoff when defining the target inflation rate for an economy with poor fiscal conditions. A lower target means a reduced welfare cost of inflation in the no-crisis zone and reduced discretionary inflation in the fiscal fragility and dominance zones, which is desirable for the policymaker. However, reducing the target also causes lower debt levels to be in the FFZ, which substantially reduces welfare since there is a positive probability that a sunspot shock that causes a permanent confidence loss will hit the economy. By computing the inflation target that maximizes total intertemporal utility for each initial debt level, we see that the optimal target is the lowest target possible such that the current debt level is in the no-crisis zone, as shown in Table III:

Insert Table III about here.

This result shows that there is a rationale for raising the inflation target in countries in a fragile fiscal situation, even when we consider the existence of costs associated with higher inflation levels. It is optimal for the policymaker to raise the target until the economy exits the FFZ. In this model, since the target is a fixed parameter, the optimal policy outside of the FFZ will be stationary, and it will not be optimal to further reduce the debt level. However, we may conjecture that in a scenario in which the inflation target is treated as a policy instrument, raising the target will be temporary, and further austerity to reduce debt levels even in the no-crisis zone will be optimal to support a lower inflation target in future periods. This is best understood as *gradual disinflation* for economies in a fragile fiscal situation.

Preference for Spending

A shock to preferences can connect our model to the situation observed in Brazil during the 2002 confidence crisis. Suppose that policymaker preferences shift toward giving more weight to public spending. Decreasing ρ would be tantamount to increasing the weight of public spending. This shift changes marginal utilities and the optimal allocation of resources, increasing the share going to public spending. The altruistic policymaker chooses higher discretionary inflation levels.

Insert Figure V about here.

Given a debt level, a relatively higher preference for public spending increases the level of discretionary inflation. For initial debt, a preference shock could push the policymaker into the FFZ. A sufficiently large shock to ρ could result in the confidence loss in the target under adverse expectations. Private agents would adapt their inflation expectations. A non-null probability assigned to an adverse event would increase expectations compared to a scenario where the target is perfectly assured. Such a preference shock explains how expectations can suddenly exceed the target, as happened in Brazil in 2002.

4 2002 Confidence Crisis in Brazil

In this section, we interpret the confidence crisis that the Brazilian government faced in the middle of 2002 through the lens of our model. In that period, Brazilian policymakers faced inflationary pressures when it became clear that the left-wing presidential candidate would win the coming elections. The perception was that his victory would mean implementing a new policy framework that could undermine the recently conquered inflation reduction. Inflation expectations overshot the target's upper bounds at all horizons relevant to the central bank, as shown in Figure I.

We map the Brazilian 2002 confidence crisis as an unforeseen shock to the preference for spending in the parameter ρ in our model. This is consistent with the perception at the time that the left-wing presidential's new framework would likely include, among other policies, a loose fiscal policy. As shown in the sensitivity analysis in Section 3.2, by favoring more public spending, the policymaker could become vulnerable to a confidence crisis that would make it deviate from the inflation target. Private agents take this probability into account when forming expectations and increase their forecasts of future inflation, precisely as observed in 2002.

The outgoing and new administrations responded to rising inflation expectations in a way that closely mirrors our model prescriptions. First, to coordinate inflation expectations in the short run, they increased the target for 2003 in an additional meeting held in June 2002 and unofficially again in January 2003. Second, during 2003, public debt reduction sustained responsible macroeconomic policies. Ultimately, inflation expectations converged back to the target. We consider each of these policies in further detail below.

Our comparative statics suggest that changing the inflation target and tax rate are good responses to avoid a self-fulfilling inflation crisis, even though the policymaker in our paper is not allowed to change the exogenously fixed tax rate and the inflation target. Also not considered by our model, the new administration faced a favorable external scenario and changed other policies, such as a new bankruptcy law, and implemented a big welfare transfer program.

Lastly, we are not claiming that the outgoing or new administrations necessarily made policy mistakes. The outgoing administration selected a debt level and inflation target consistent with their policy and preferences. Evidence is that inflation expectations were within the target bounds for all the relevant horizons before the elections. The new administration inherited a debt level and inflation target that may have been inconsistent with their preference parameters, potentially pushing the economy into the fiscal fragility zone. However, policy responses seemed to be the right ones through the lens of our model.

Inflation Target

As mentioned, the 2003 target was exceptionally revised upward before the October elections in response to the surge in inflation expectations. It went from a previously announced 3.25% to 4%, and the upper and lower bounds widened from -/+ 2% to 2.5%. The target was again revised in January 2003, when the Ministry of Finance sent a letter stating that the adjusted target would be 8.5% in 2003 and 5.5% in 2004 – the National Monetary Committee confirmed the inflation target for 2004 in June 2003. Those changes can be seen in Table IV.

An indebted economy with a higher inflation target is more likely to be outside the fiscal fragility zone and within the zone where private agents believe the policymaker will deliver the target. The higher inflation target serves as a nominal anchor, making private agents readjust their inflation expectations. From the perspective of our model, this change in the target was a strategy by the Brazilian government to anchor inflation expectations.

Insert Table IV about here.

Fiscal Policy

After the 2002 election, the government gradually reduced the gross public debt. The gross debt declined from nearly 80% of GDP in 2002 to nearly 70% in 2004. Furthermore, the government continued to run primary surpluses to meet its debt obligations in a signal of fiscal responsibility. The primary surplus increased from 2.16% of GDP in 2001 to 2.70% in 2004. From the perspective of our model, debt reduction is compatible with the policymaker's attempts to exit the fiscal fragility zone and provide the necessary fiscal support to achieve its inflation target.

5 An Extension with Real Debt

Many countries, especially emerging economies, issue a mix of nominal and real debt.[17] We consider an extension of our model in which real debt is available, a form of public debt in which the government cannot default via surprise inflation. Real debt can be interpreted as either foreign-currency debt, assuming that the price level moves one-to-one with the exchange rate or inflation-indexed bonds. We also assume that the real-debt fraction is not subject to any form of default. If we consider the possibility that the policymaker may not honor its obligations to real-debt holders, we fall into similar questions as the ones analyzed by the sovereign debt literature, such as Eaton and Gersovitz (1981), Cole and Kehoe (2000), Arellano (2008), Aguiar and Gopinath (2006), and others.

We show two results of introducing non-defaultable real debt in our model. First, the government deviates to a lower inflation rate, given the smaller rollover-costs channel. Second, the policymaker can avoid the fiscal fragility zone for any initial debt level by issuing a sufficiently high fraction of non-defaultable real debt. However, this fraction may no longer be compatible with the hypothesis of fully committed real debt contracts.

Let $D_t = D_t^N + D_t^F$, where D_t^N is debt issued in domestic currency subject to a partial default via inflation, and D_t^F is real debt. Due to the assumed risk neutrality, private agents are indifferent between both bonds so long as they have the same expected return of $1/\beta$. Since real debt always delivers the same return in every state, the real rate over D_t^F in equilibrium is simply $1/\beta$, while the real return over D_t^N is given by the same relations (3) and (6).

With this specification, the policymaker's discretionary deviation problem changes since it can only default on the nominal debt. Let n_t represent the fraction of nominal debt so that $D_t^N = n_t D_t$ and $D_t^F = (1 - n_t) D_t$. The policymaker problem of deviating in period T, given an initial debt level D_T and nominal fraction n_T is:

[17] In October 2002, 63% of the Brazilian public was nominal denominated and subjected to partial default, while 37% was denominated in dollars or indexed to the inflation index. In October 2023, these numbers were 68% and 32%, respectively.

$$\max_{\pi,D} u(c_T, g_T) + \frac{\beta}{1-\beta} u(c, g) - \frac{1}{1-\beta}\psi(\pi)$$

subject to

$$\begin{aligned} g_T &= \tau(e-\epsilon) - n_T(1+r_T^D)D_T + \frac{1-n_T}{\beta}D_T + D \\ g &= \tau(e-\epsilon) - D\left(\frac{1}{\beta} - 1\right) \\ c_T &= e - \epsilon - g_T \\ c &= e - \epsilon - g \\ 1 + r_T^D &= \frac{1}{\beta}\frac{1}{q_T^e}\frac{1}{1+\pi} \ . \end{aligned} \tag{17}$$

After deviating, the economy enters a steady state, and the nominal/real mix is irrelevant since nominal debt return also becomes perfectly anticipated. The solution to the policymaker problem is now given by an optimal stationary debt level:

$$D = \beta n_T(1 + r_T^D)D_T + (1 - n_T)D_T \ , \tag{18}$$

and the optimal discretionary inflation is given by the FOC:

$$n_T(\rho - (1-\rho)v'(g))\frac{\partial(1 + r_T^D)}{\partial \pi}D_T - \frac{1}{1-\beta}\psi'(\pi) = 0 \ . \tag{19}$$

This is the same FOC of the original problem but with a smaller term associated with the marginal utility of inflation, given the fraction $n_T \in [0, 1]$ multiplying the first term. As a result, discretionary inflation is lower in this model specification. Also, we see that the optimal discretionary inflation is simply $\pi = 0$ if $n_T = 0$.[18]

Given an initial debt level D and nominal fraction n, the policymaker chooses whether to deviate to the discretionary inflation level, next period debt D', and nominal fraction n'. Now, the conditions that characterize the three different scenarios over the debt level must be checked for each pair (D, n). If we assume that ϵ is sufficiently high so that deviating is not essentially costless to the policymaker, we can reestablish Proposition 3 in this formulation and conclude that the economy is in the no-crisis zone when $n = 0$. This happens because, in this case, the discretionary inflation equals zero and is, therefore, lower than the target π^a. We can establish the following proposition:

[18]Formally, this requires adding the natural assumption that either $\psi'(0) = 0$ or the restriction $\pi \geq 0$. The first restriction implies that there is a global ex-ante optimal inflation equal to zero. The second restriction can be necessary if we consider, for example, a linear specification over ψ, in which case there would be no solution for the problem without an additional restriction over the set of π.

Proposition 6 ***(Escaping the FFZ by Issuing Real Debt)***: *Given an initial debt level D, there exists a sufficiently low fraction of nominal debt $n^D \geq 0$ such that (D, n^D) is in the no-crisis zone.*

This proposition follows from $(D, 0)$ being in the no-crisis zone and V^p, the functions used to check the no-crisis zone condition, being continuous. Given any initial debt level D, it implies that the government can simply exit the FFZ by setting n' sufficiently low and limiting its ability to use inflation to reduce roll-over costs. In other words, the policymaker can increase confidence that it will deliver the target by raising a fraction of debt in real terms and reducing the incentive to deviate from the target. However, in the scenario of no perfect commitment to honor foreign currency contracts, the policymaker can enter a region of distrust over its commitment to repay the foreign debt holders if the fraction of real debt is sufficiently high, as argued in Cole and Kehoe (2000).

6 Concluding Remarks

In this paper, we develop a model to study the problem of setting the inflation target in an economy with a high debt burden. In light of our results, we can analyze a concrete case in which the Brazilian economy was subject to an adverse inflation expectations shock and where the policymakers successfully avoided a severe loss of confidence in the monetary regime by simultaneously raising the inflation target and adopting fiscal austerity measures. Our results provide practical implications for the design of monetary policy frameworks for economies with fragile fiscal fundamentals. Adopting a low inflation target may not be welfare-improving if it raises doubts over the commitment of the policymaker to delivering the target.

Appendix A Proofs

A.1 Discretionary Inflation is Increasing in the Debt Level

Suppose that π_T^D is an interior solution to the discretionary inflation problem. To avoid a cumbersome notation we drop the superscript on r_T^D and let

$$1 + r_T = \frac{1}{\beta} \frac{1}{f + (1-f)\frac{1+\pi}{1+\pi^a}} \tag{20}$$

be the real interest rate when the policymaker deviates to inflation level π, where f is the probability of a negative sunspot shock. To obtain $\partial \pi_T^D / \partial D_T$, we differentiate the first-order condition (11) with respect to the initial debt level D_T to obtain:

$$\begin{aligned} -(1-\rho)v''(g)\frac{\partial g}{\partial D_T}\frac{\partial(1+r_T)}{\partial \pi}D_T + (\rho - (1-\rho)v'(g))\frac{\partial^2(1+r_T)}{\partial \pi \partial D_T}D_T \\ +(\rho - (1-\rho)v'(g))\frac{\partial(1+r_T)}{\partial \pi} = \frac{1}{1-\beta}\psi''(\pi)\frac{\partial \pi}{\partial D_T} \end{aligned} \tag{21}$$

where

$$\frac{\partial g}{\partial D_T} = -\beta \frac{\partial(1+r_T)}{\partial \pi}\frac{\partial \pi}{\partial D_T}D_T - \beta(1+r_T) \tag{22}$$

and

$$\frac{\partial^2(1+r_T)}{\partial \pi \partial D_T} = \frac{\partial^2(1+r_T)}{\partial \pi^2}\frac{\partial \pi}{\partial D_T} \tag{23}$$

since $1 + r_T$ does not depend directly on D_T.

To prove that $\partial \pi_T^D / \partial D_T > 0$, we substitute back equations (22) and (23) in (21) and rearrange to obtain:

$$\begin{aligned} \left[\frac{1}{1-\beta}\psi''(\pi) - \beta(1-\rho)v''(g)\left(\frac{\partial(1+r_T)}{\partial \pi}\right)^2 D_T^2 - [\rho - (1-\rho)v'(g)]\frac{\partial^2(1+r_T)}{\partial \pi^2}\right]\frac{\partial \pi}{\partial D_T} \\ = [\rho - (1-\rho)v'(g) + \beta(1-\rho)v''(g)(1+r_T)D_T] \end{aligned} \tag{24}$$

By differentiating (20) it is easy to conclude that $\frac{\partial(1+r_T)}{\partial \pi} < 0$ and $\frac{\partial^2(1+r_T)}{\partial \pi^2} > 0$. Also, by hypothesis, we have that for every feasible g, $\rho - (1-\rho)v'(g) < 0$ and $v''(g) < 0$. Since both the term multiplying $\frac{\partial \pi}{\partial D_T}$ and the term on the right hand side in equation (24) are strictly positive, we conclude that $\frac{\partial \pi}{\partial D_T} > 0$.

A.2 Optimal Debt Policy Outside the Fiscal Fragility Zone

Outside of the FFZ, there is only a unique inflation equilibrium, making it perfectly anticipated. The policymaker's problem can be reduced to the following:

$$\begin{aligned} \max_{D_{t+1}} \sum_{t \geq S} & \beta^t u(c_t, g_t) \\ \text{s.t. } c_t &= \frac{1}{\beta} D_t + (1-\tau)(e - \epsilon^*) - D_{t+1} \\ g_t &= D_{t+1} + \tau(e - \epsilon^*) - \frac{1}{\beta} D_t \end{aligned}$$

where ϵ^* is equal to 0 if the economy is in the no-crisis zone and equal to ϵ if it is in the fiscal dominance zone. The first-order condition (FOC) for D_{t+1} yields:

$$(1-\rho)v'(g_t) - \rho = (1-\rho)v'(g_{t+1}) - \rho$$

which implies $v'(g_t) = v'(g_{t+1})$ for all t. Given that v strictly concave in g, we must have $g_{t+1} = g_t$. Replacing g_t and g_{t+1} by the government budget equation, iterating forward and taking limits, we obtain:

$$\lim_{t \to \infty} D_{S+t} = \sum_{i=1}^{\infty} \left(\frac{1}{\beta}\right)^i (D_{S+1} - D_S) + D_{S+1}. \tag{25}$$

Suppose that $D_{S+1} \neq D_S$; then, the policymaker will either run up infinite debt or credit. Equation (25) also implies that

$$\lim_{t \to \infty} \beta^{S+t-1} D_{S+t} = \beta^S \sum_{i=0}^{\infty} \beta^i (D_{S+1} - D_S) + \lim_{t \to \infty} \beta^{S+t-1} D_{S+1} = \frac{\beta^S}{1-\beta}(D_{S+1} - D_S).$$

The no-Ponzi condition for this problem states that

$$\lim_{t \to \infty} \beta^t D_{t+1} = 0,$$

so that if $D_{S+1} \neq D_S$, this condition is violated. This means that the only optimal trajectory for debt outside of the FFZ is the stationary state such that $D_t = D_S = D_{S+1}$ for all t.

A.3 Above-Target Discretionary Inflation

The intuition for the proof is simple: we show that if it is the case that $\pi^D < \pi^a$, then there exists a feasible policy that does not deviate from the target and attains a higher intertemporal utility then deviating, even if private agents believe that the policymaker will deviate from target. This implies that the economy is in the no-crisis zone. Therefore, whenever the economy is in the FFZ, deviation from target must be positive.

For a given debt level D, assume that $\pi^D < \pi^a$, that is, the discretionary inflation rate is lower than the target rate. We want to prove that in this case, $V^p(D, \pi_t = \pi^a, \pi^e = \pi^D) \geq V^p(D, \pi_t = \pi^D, \pi^e = \pi^D)$, that is, the policymaker follows the inflation target even when the private agents expect the policymaker to deviate. Let T be the time of deviation, and assume that private agents expect the policymaker to deviate; then, according to Equation (7), total government spending both in period T and in the stationary long-run will be equal to

$$g^D = \tau(e - \epsilon) - \left(\frac{1}{\beta} - 1\right) D, \tag{26}$$

since agents expect the deviation.

As an alternative, the government can choose the feasible path of not deviating from the target and following a stationary spending policy:

$$g^a = \tau e - \left(\frac{1 + \pi^D}{1 + \pi^a} \frac{1}{\beta}\right) D. \tag{27}$$

Since $\pi^D < \pi^a$, we have that $g^D < g^a$, as

$$g^a - g^D = \tau\epsilon + \frac{\pi^a - \pi^D}{1 + \pi^a} \frac{1}{\beta} D > 0.$$

To compare the total intertemporal utility of the two policies, we only need to compare which one of the allocations achieves a higher utility in any period, since they are stationary allocations. Let $c^D = e - \epsilon - g^D$ and $c^a = e - g^a$ be the market-clearing private consumption in each scenario. By the concavity of the utility function, we have that

$$\begin{aligned} u(c^D, g^D) - u(c^a, g^a) \leq & \rho(c^D - c^a) + (1 - \rho)v'(g^a)(g^D - g^a) \\ = & -\rho\epsilon + ((1 - \rho)v'(g^a) - \rho)(g^D - g^a) \\ < & -\psi(\pi^a) < \psi(\pi^D) - \psi(\pi^a) \end{aligned} \tag{28}$$

since, by the assumption, we have that $(1 - \rho)v'(g^a) - \rho \geq 0$, $\rho\epsilon > \psi(\pi^a)$, and $\pi^D \leq \pi^a$.

Now, since there is a feasible policy trajectory in which the policymaker follows the inflation target and its intertemporal utility is greater than that attained by deviating to the inflation rate π^D, this means that the optimal policy chosen by the policymaker when following the target must also attain a higher utility than the attained by deviating, that is, $V^p(D, \pi_t = \pi^a, \pi^e = \pi^D) \geq V^p(D, \pi_t = \pi^D, \pi^e = \pi^D)$. However, this means that the policymaker chooses to follow the target even when private agents expect it to deviate, so that debt level D is in the no-crisis zone.

A.4 Real Interest Rates in the FFZ when Raising the Target

By the definition of the real interest rate, when the policymaker follows the target in the FFZ we have

$$1 + r_t^a = \frac{1}{\beta} \frac{1}{f \frac{1+\pi^a}{1+\pi} + 1 - f}. \tag{29}$$

Differentiating with respect to π^a and we obtain

$$\frac{\partial(1 + r_t^a)}{\partial \pi^a} = \frac{1}{\beta} \frac{1}{\left[f \frac{1+\pi^a}{1+\pi} + 1 - f\right]^2} \frac{f}{1+\pi} \left(\frac{1+\pi^a}{1+\pi} \frac{\partial \pi}{\partial \pi^a} - 1 \right) \tag{30}$$

so that the desired result is shown if the term in parenthesis is positive. To prove that, we calculate $\partial\pi/\partial\pi^a$ implicitly by differentiating equation (11)[19] with respect to π^a, obtaining:

$$(1-\rho)v''(g)(1-\beta)\frac{\partial(1+r_t)}{\partial \pi^a}\frac{\partial(1+r_t)}{\partial \pi}D_T^2 + (\rho-(1-\rho)v'(g))\frac{\partial^2(1+r_t)}{\partial\pi\partial\pi^a}D_T = \frac{1}{1-\beta}\left[\psi''\frac{\partial \pi}{\partial \pi^a} + \frac{\partial \psi'}{\partial \pi^a}\right] \tag{31}$$

From the definition of $1 + r_t$, we can obtain the following identities:

$$\begin{aligned}
\frac{\partial(1+r_t)}{\partial \pi^a} &= \frac{\partial(1+r_t)}{\partial \pi}\left[\frac{\partial \pi}{\partial \pi^a} - \frac{1+\pi}{1+\pi^a}\right] \\
\frac{\partial^2(1+r_t)}{\partial\pi\partial\pi^a} &= \frac{\partial(1+r_t)}{\partial \pi}\left[2\beta\frac{1-f}{1+\pi^a}(1+r_t)\left(\frac{1+\pi}{1+\pi^a} - \frac{\partial \pi}{\partial \pi^a}\right) - \frac{1}{1+\pi^a}\right]
\end{aligned}$$

[19] We again drop the superscript on r_T^D to avoid cumbersome notation.

which we can substitute back to find the solution for $\partial\pi/\partial\pi^a$, given by:

$$\left[2\beta\frac{1-f}{1+\pi^a}(1+r_t)(\rho-(1-\rho)v'(g))D_T\frac{\partial(1+r_t)}{\partial\pi}-(1-\beta)(1-\rho)v''(g)D_T^2\left(\frac{\partial(1+r_t)}{\partial\pi}\right)^2\right.$$
$$\left.+\frac{1}{1-\beta}\psi''\right]\frac{\partial\pi}{\partial\pi^a}=-\frac{1}{1-\beta}\frac{\partial\psi'}{\partial\pi^a}-(1-\beta)(1-\rho)v''(g)D_T^2\left(\frac{\partial(1+r_t)}{\partial\pi}\right)^2\frac{1+\pi}{1+\pi^a}$$
$$+(\rho-(1-\rho)v'(g))D_T\frac{\partial(1+r_t)}{\partial\pi}\left[2\beta\frac{1-f}{1+\pi^a}(1+r_t)\frac{1+\pi}{1+\pi^a}-\frac{1}{1+\pi^a}\right]$$

The first term in brackets is clearly strictly positive so that $\partial\pi/\partial\pi^a$ is well defined. Using the result above we can calculate the term in parenthesis in equation (30), which results in a fraction with the same (strictly positive) denominator as $\partial\pi/\partial\pi^a$, and numerator given by:

$$-\frac{1}{1-\beta}\left[\psi''+\frac{1+\pi^a}{1+\pi}\frac{\partial\psi'}{\partial\pi^a}\right]-(\rho-(1-\rho)v'(g))\frac{\partial(1+r_t)}{\partial\pi}\frac{1}{1+\pi}D_T<0$$

which proves that $\frac{\partial(1+r_t^a)}{\partial\pi^a}<0$. An analogous calculation shows that $\frac{\partial(1+r_t^D)}{\partial\pi^a}>0$, and we are done.

A.5 Lower Limit of the Fiscal Fragility Zone

We start by characterizing the debt level $\underline{D}$ that is the lower limit of the FFZ. Consider a D in the no-crisis zone, then:

$$\begin{aligned}V^p(D,\pi_t=\pi^a,\pi^e=\pi^D)=&u\left((1-\tau)e+\left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a}-1\right)D,\tau e-\left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a}-1\right)D\right)-\psi(\pi^a)\\&+\beta V^p(D,\pi^e,\zeta=0,\epsilon_{-1}=0)\end{aligned}$$

Since for every debt level in the no-crisis zone it is optimal to keep the debt level constant in every period by Proposition 2, we know that $\pi^e=\pi^a$ and

$$V^p(D,\pi^e,\zeta=0,\epsilon_{-1}=0)=\frac{1}{1-\beta}\left[u\left((1-\tau)e+\left(\frac{1}{\beta}-1\right)D,\tau e-\left(\frac{1}{\beta}-1\right)D\right)-\psi(\pi^a)\right]$$

Then, for such D, the no-crisis zone condition is satisfied with

$$\begin{aligned}
V^p(D,\pi_t=\pi^a,\pi^e=\pi^D) =& u\left((1-\tau)e+\left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a}-1\right)D, \tau e-\left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a}-1\right)D\right)-\psi(\pi^a)\\
&+\frac{\beta}{1-\beta}\left[u\left((1-\tau)e+\left(\frac{1}{\beta}-1\right)D, \tau e-\left(\frac{1}{\beta}-1\right)D\right)-\psi(\pi^a)\right]\\
\geq& \frac{1}{1-\beta}\left[u\left((1-\tau)(e-\epsilon)+\left(\frac{1}{\beta}-1\right)D, \tau(e-\epsilon)-\left(\frac{1}{\beta}-1\right)D\right)-\psi(\pi^D)\right]\\
=& V^p(D,\pi_t=\pi^D,\pi^e=\pi^D)
\end{aligned}$$

Since π^D is continuous with respect to D, we have that in the point $\underline{\mathrm{D}}$ the condition above is satisfied with equality, otherwise there would exist a $D>\underline{\mathrm{D}}$ sufficiently close that satisfies the above condition and so is also in the no-crisis zone. Let $\underline{\mathrm{D}}$ be the point where the condition is satisfied with equality, so that

$$\begin{aligned}
&u\left((1-\tau)e+\left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a}-1\right)\underline{\mathrm{D}}, \tau e-\left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a}-1\right)\underline{\mathrm{D}}\right)-\psi(\pi^a)\\
&+\frac{\beta}{1-\beta}\left[u\left((1-\tau)e+\left(\frac{1}{\beta}-1\right)\underline{\mathrm{D}}, \tau e-\left(\frac{1}{\beta}-1\right)\underline{\mathrm{D}}\right)-\psi(\pi^a)\right]\\
&=\frac{1}{1-\beta}\left[u\left((1-\tau)(e-\epsilon)+\left(\frac{1}{\beta}-1\right)\underline{\mathrm{D}}, \tau(e-\epsilon)-\left(\frac{1}{\beta}-1\right)\underline{\mathrm{D}}\right)-\psi(\pi^D)\right]
\end{aligned}$$

We can rewrite the above expression as

$$u\left((1-\tau)e+\left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a}-1\right)\underline{\mathrm{D}}, \tau e-\left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a}-1\right)\underline{\mathrm{D}}\right)-\frac{1}{1-\beta}\left[\psi(\pi^a)-\psi(\pi^D)\right] \tag{32}$$

$$\begin{aligned}
=&\frac{1}{1-\beta}u\left((1-\tau)(e-\epsilon)+\left(\frac{1}{\beta}-1\right)\underline{\mathrm{D}}, \tau(e-\epsilon)-\left(\frac{1}{\beta}-1\right)\underline{\mathrm{D}}\right)\\
&-\frac{\beta}{1-\beta}u\left((1-\tau)e+\left(\frac{1}{\beta}-1\right)\underline{\mathrm{D}}, \tau e-\left(\frac{1}{\beta}-1\right)\underline{\mathrm{D}}\right)
\end{aligned}$$

in which only the left-hand side varies when we change the inflation target π^a. We now prove that the left-hand side of the above expression is increasing in π^a, which implies that for a higher target the no-crisis zone condition is satisfied with inequality for $\underline{\mathrm{D}}$. This means that there exists a $D>\underline{\mathrm{D}}$ in the no crisis zone, and therefore the lower limit of the fiscal fragility zone is higher under the higher inflation target.

Differentiate the left hand side of equation (32) with respect to π^a to obtain

$$LHS'(\pi^a) = (\rho-(1-\rho)v'(g_a^D))\frac{1}{1+\pi^a}\left(\frac{\partial \pi^D}{\partial \pi^a} - \frac{1+\pi^D}{1+\pi^a}\right)\underline{\mathrm{D}} - \frac{1}{1-\beta}\left(\psi'(\pi^a) - \psi'(\pi^D)\frac{\partial \pi^D}{\partial \pi^a}\right) \tag{33}$$

where $g_a^D = \tau e - \left(\frac{1}{\beta}\frac{1+\pi^D}{1+\pi^a} - 1\right)\underline{\mathrm{D}}$ is government spending when delivering the target given expectations to deviate. We substitute the expression for $\frac{\partial \pi^D}{\partial \pi^a}$ from the proof of Proposition 4, to obtain

$$\begin{aligned}
&\left[\frac{1}{1-\beta}\psi''(\pi^D) + 2\beta\frac{1-f}{1+\pi^a}(1+r)(\rho-(1-\rho)v'(g))\underline{\mathrm{D}}\frac{\partial(1+r)}{\partial \pi}\right.\\
&\qquad\left. -(1-\beta)(1-\rho)v''(g)\underline{\mathrm{D}}\left(\frac{\partial(1+r)}{\partial \pi}\right)^2\right]LHS'(\pi^a)\\
&= \left[\frac{1+\pi^D}{1+\pi^a}\psi'(\pi^D) - \psi'(\pi^a)\right]\left(2\frac{\beta}{1-\beta}\frac{1-f}{1+\pi^a}(1+r)(\rho-(1-\rho)v'(g))\underline{\mathrm{D}}\frac{\partial(1+r)}{\partial \pi}\right.\\
&\qquad\left. -(1-\rho)v''(g)\underline{\mathrm{D}}^2\left(\frac{\partial(1+r)}{\partial \pi}\right)^2\right)\\
&- \left((\rho-(1-\rho)v'(g))\frac{\underline{\mathrm{D}}}{1+\pi^a}\frac{\partial(1+r)}{\partial \pi} + \frac{1}{1-\beta}\frac{\partial \psi'}{\partial \pi^a}\right)\left[(\rho-(1-\rho)v'(g_a^D))\frac{\underline{\mathrm{D}}}{\beta(1+\pi^a)} + \frac{1}{1-\beta}\psi'(\pi^D)\right]\\
&\qquad -\frac{1}{1-\beta}\psi''(\pi^D)\frac{1+\pi^D}{1+\pi^a}\left[(\rho-(1-\rho)v'(g_a^D))\frac{\underline{\mathrm{D}}}{\beta(1+\pi^a)} + \frac{1}{1-\beta}\psi'(\pi^a)\frac{1+\pi^a}{1+\pi^D}\right]
\end{aligned} \tag{34}$$

where g is government spending solution and r is the real interest rate in the optimal deviation problem. The sign of $LHS'(\pi^a)$ is determined by the sign of the term in the right hand side of (34), since as in the proof of Proposition 4 the denominator of $\frac{\partial \pi^D}{\partial \pi^a}$ is positive.

We start by showing that

$$\begin{aligned}
&\left[\frac{1+\pi^D}{1+\pi^a}\psi'(\pi^D) - \psi'(\pi^a)\right]\left(2\frac{\beta}{1-\beta}\frac{1-f}{1+\pi^a}(1+r)(\rho-(1-\rho)v'(g))\underline{\mathrm{D}}\frac{\partial(1+r)}{\partial \pi}\right.\\
&\qquad\left. -(1-\rho)v''(g)\underline{\mathrm{D}}^2\left(\frac{\partial(1+r)}{\partial \pi}\right)^2\right) > 0
\end{aligned}$$

Using the same analysis as in the proofs of Propositions 1 and 4 we know that the term in parenthesis is strictly positive. We also have that

$$\frac{1+\pi^D}{1+\pi^a}\psi'(\pi^D) - \psi'(\pi^a) > 0$$

is immediate from the hypothesis that $\psi'' \geq 0$, and the fact that $\pi^D > \pi^a$. This is true since we know from Proposition 3 that if $\pi^D \leq \pi^a$ then the no-crisis zone condition is satisfied with strict inequality, and in point $\underline{\mathrm{D}}$ it is satisfied with equality.

We now prove that the rest of the right hand side of 34 is positive. The previous reasoning also imply that

$$(\rho-(1-\rho)v'(g_a^D))\frac{\underline{\mathrm{D}}}{\beta(1+\pi^a)}+\frac{1}{1-\beta}\psi'(\pi^a)\frac{1+\pi^a}{1+\pi^D} < (\rho-(1-\rho)v'(g_a^D))\frac{\underline{\mathrm{D}}}{\beta(1+\pi^a)}+\frac{1}{1-\beta}\psi'(\pi^D)$$

so that we only need to show that

$$\begin{aligned}-\left((\rho-(1-\rho)v'(g))\frac{\underline{\mathrm{D}}}{1+\pi^a}\frac{\partial(1+r)}{\partial\pi}+\frac{1}{1-\beta}\left(\frac{1+\pi^D}{1+\pi^a}\psi''(\pi^D)+\frac{\partial\psi'}{\partial\pi^a}\right)\right)\\ \left[(\rho-(1-\rho)v'(g_a^D))\frac{\underline{\mathrm{D}}}{\beta(1+\pi^a)}+\frac{1}{1-\beta}\psi'(\pi^D)\right]>0\end{aligned}$$

Again, the fact that the term in parenthesis is strictly positive was proved in Proposition 4. We only need to prove that the term in braces is strictly negative. Using the first order condition (11) and the definition for $1+r$ we get

$$\begin{aligned}\frac{1}{1-\beta}\psi'(\pi^D) =&(\rho-(1-\rho)v'(g))\frac{\partial(1+r)}{\partial\pi}\underline{\mathrm{D}}\\ =&((1-\rho)v'(g)-\rho)\frac{\underline{\mathrm{D}}}{\beta(1+\pi^a)}\frac{(1-f)}{\left(f+(1-f)\frac{1+\pi^D}{1+\pi^a}\right)^2}\\ <&((1-\rho)v'(g_a^D)-\rho)\frac{\underline{\mathrm{D}}}{\beta(1+\pi^a)}\end{aligned}$$

so long as we show that $g_a^D < g$, since $0 < \frac{(1-f)}{\left(f+(1-f)\frac{1+\pi^D}{1+\pi^a}\right)^2} < 1$. But from expression (32), we know that

$$u(e-g_a^D,g_a^D)+\frac{\beta}{1-\beta}u(e-g_a^a,g_a^a)-\frac{1}{1-\beta}\psi(\pi^a)=\frac{1}{1-\beta}u(e-\epsilon-g_D^D,g_D^D)-\frac{1}{1-\beta}\psi(\pi^D)$$

where $g_a^a=\tau e-(\frac{1}{\beta}-1)\underline{\mathrm{D}}$ and $g_D^D=\tau(e-\epsilon)-(\frac{1}{\beta}-1)\underline{\mathrm{D}}$. Since $g_a^a > g_D^D$ and $\psi(\pi^D) > \psi(\pi^a)$, the only way for the above equality to be satisfied is for $g_a^D < g_D^D$, since u is increasing in government spending. However, it is straightforward to show that $g_D^D < g$, since g is the stationary spending when the government is able to partially default in debt and reduce interest spending. But this proves that $g_a^D < g$ and we are done.

Appendix B Empirical Results

The calibrated model leads to the conclusions that i) the size of the deviation could be reduced by increasing the target and reducing debt and ii) the probability of deviating from the target would increase with debt and decrease with higher target levels. The present section investigates whether there is empirical evidence for the predictions based on our model. We construct a dataset that includes 20 countries with at least 15 years of inflation targeting[20] covering the period from 2000 to 2019. Targets are those reported by the respective central banks that were manually collected from each central bank web page. Inflation and gross debt and revenue to GDP statistics are from the IMF. With regard to inflation, end-of-year consumer price inflation is the target benchmark. Some general statistics are reported in Table B.I. The variables present both inter- and intracountry variability. In the case of CPI targets, 55% of our sample changed the target at least once. Most of the changes are in middle-income countries.[21]

Insert Table B.I about here.

Real effective exchange rate (Reer) and GDP gap estimates enter robustness checks. When Reer statistics were not available from the IMF, other sources were accessed.[22] GDP gap estimates are constructed using quarterly seasonally adjusted GDP volume statistics from the IMF. When not available, the unadjusted equivalents are seasonally adjusted with the Arima X-11 procedure.[23] The quarterly GDP gap statistics are obtained applying an HP filter with a smoothing parameter of 1600. To mitigate the endpoint bias of the filter at the beginning of each series, we estimate the gap for the longer 1996Q1 - 2020Q1 period. Finally, the yearly GDP gap is defined as the average gap over the relevant period.

Deviations from the Target

The FOC of the discretionary inflation problem from 7 relates the deviation of inflation $\pi_{i,t}$ from the inflation target $\pi^a_{i,t}$ to observable and latent variables for each country i. We estimate the following model,

$$\pi_{i,t} - \pi^a_{i,t} = \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \pi^a_{i,t} + \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t} \tag{35}$$

[20] The countries in the sample are Australia, Brazil, Canada, Chile, Colombia, the Czech Republic, Iceland, Indonesia, Israel, Mexico, New Zealand, Norway, Peru, the Philippines, Poland, South Africa, Sweden, Thailand, Turkey, and the United Kingdom.

[21] We used the World Bank classification.

[22] BIS for Peru, Indonesia, and Turkey. Bank of Thailand for Thailand.

[23] This was the case for Peru and Turkey.

where the idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1},...,X_{i,T},c_i) = 0,\ t = 1,...,T$ with $X_{i,t}$ being a vector of the observable regressors at time t for country i. The variables and parameters of the model are mapped into both observed series and latent variables. We map the model variables D, τe, and π^a to gross debt (%GDP), revenue (%GDP), and the inflation target. The unobservable variables e, f, ϵ and κ are mapped into a country fixed effect c_i that captures the time-constant individual heterogeneity between countries. We use a fixed effect estimator because it seems reasonable to assume that their choices of debt, revenue and inflation target are related to the unobserved characteristics of each country c_i. In other words, we cannot assume $\mathbb{E}(X_{i,t}c_i) = 0\ \forall t$ as required for a random effect estimator.[24]

In terms of interpretation, the net impact of debt should be positive. Given higher levels of debt, the policymaker will have more incentive for discretionary inflation. Furthermore, discretionary inflation increases in debt. Hence, the deviation to increase in debt levels as the policymaker will be more likely to deviate and will choose higher discretionary inflation when doing so. Given an interaction term in (35), one would have to examine the joint impact captured by β_2 and β_4 for a given level of revenue to GDP. We also expect the coefficient on the inflation target to be negative because the policymaker could help coordinate private agents' expectations by adopting a more believable (higher) inflation target in given situations. Were inflation perfectly anchored, changing the target would not result in changes in expected deviation. In other words, the coefficient β_3 would equal zero. Finally, higher revenue means that the policymaker has more fiscal room for spending. This room decreases the incentives to transfer resources through discretionary inflation, leading to a negative net impact of revenue. Given the interaction term between debt and revenue, the joint impact captured by β_1 and β_4 should be negative for a given level of debt.

Estimation I in Table B.II is the basic model from (35). The remaining estimations, II-V, are robustness checks.

Insert Table B.II about here.

In estimation I, deviations from the target are on average negatively related to the target level. In the case of perfectly anchored inflation, the coefficient should not be significantly different from zero. We also have a positive coefficient on debt and a negative coefficient for the interaction term between debt and revenues. This can be interpreted as higher debt implying higher deviations for countries with limited revenues. For revenues no higher than 35% of GDP, the net impact of debt is positive. This result applies to the middle-income countries in our sample. The result goes in the direction of what the theoretical model predicted,

[24] A Hausman test between a fixed and random effect estimator similarly suggests the use of the former.

as both the probability of deviating and deviations from the target are positively related to debt levels. On average, countries with higher debt levels have higher deviations from their inflation target.

The coefficient on revenue is positive in all settings although not always significant. Given the interaction term with debt, the net impact of revenue is positive up to debt levels of 88%, above the maximum in our sample. Hence, the impact of higher revenue is to increase deviations from the inflation target. Although this goes against what was expected from the theoretical model, one could argue that higher revenue could be correlated with preferences for public spending that in turn could lead to inflationary pressure.

The results remain after accounting for different types of shocks and variables usually associated with inflation dynamics. In estimation II, we include a time fixed effect to account for global shocks such as commodity prices. In our sample, 2008 stands out, as many countries overshot their inflation targets after the financial crisis. The time dummies are meant to take such global comovements in inflation into account. Estimation III also includes shocks to the real effective exchange rate. Estimation IV adds the impact of deviations from potential GDP on inflation.

Probability of Deviating from the Target

The policymaker deviates from the inflation target when end-of-year inflation exceeds the upper bound of the target.[25] In the theoretical model, the policymaker had more incentive to deviate when it had limited fiscal space due to high debt servicing cost. We estimate a similar equation to (35) but with regard to the probability of deviating:

$$I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = \beta_1 \text{revenue}_{i,t} + \beta_2 \text{debt}_{i,t} + \beta_3 \text{target}_{i,t} + \beta_4 \text{revenue}_{i,t} * \text{debt}_{i,t} + c_i + u_{i,t} \tag{36}$$

where $\overline{\pi}^A_{i,t}$ is the upper bound of the inflation target for country i at time t. The indicator $I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = 1$ when inflation $\pi_{i,t}$ exceeds the upper bound of the inflation target $\overline{\pi}^A_{i,t}$. The idiosyncratic error $u_{i,t}$ satisfies $\mathbb{E}(u_{i,t}|X_{i,1},...,X_{i,T},c_i) = 0,\ t = 1,...,T$. The probability of deviating will then be a logistic function:

$$Pr(I_{\pi_{i,t}>\overline{\pi}^A_{i,t}} = 1|X_{i,t}, c_i) = \frac{1}{1+e^{-X'_{i,t}\beta - c_i}}, \quad t = 1,...,T \tag{37}$$

[25] Some countries adopt pointwise targets instead of tolerance bounds. This is for instance the case for the UK and Norway. In such cases, we used the average upper tolerance limit from the rest of the sample (1.2%).

The expected results and dynamics are quite similar to those in the previous section with an expected net positive impact of debt, negative impact of the inflation target, and negative impact of revenue on the probability of deviating. Each year in the sample, at least two countries deviate from their respective inflation targets. The years 2007 and 2008 stand out, as over half of the countries deviated. A time dummy is likely to capture this effect. Additionally, virtually all countries except two deviated from their targets at least once, with some countries such as Turkey close to being serial deviators. Overall, middle-income countries exceed the target more often than high-income countries. Nevertheless, high-income countries exceeded the target 39 times.

The first column of Table B.III is the baseline model, while the remaining columns represent robustness checks similar in spirit to the previous section. When considering the net impact of debt on the probability of deviating from the target, the coefficients have similar signs to the previous estimates with regard to deviations from the target. Estimation I has the most restrictive condition for a net positive effect of debt. For revenues over 30% of GDP, the net effect of debt stops being positive. Not all middle-income countries in our sample have revenue below this level. However, the effects are not statistically significant in any of the settings.

Insert Table B.III about here.

The net impact of revenue remains positive for debt levels in the sample, not in the same direction as predicted by the theoretical model. The predicted negative impact of debt is based on increased fiscal room provided by higher revenue, which decreases the incentive to use discretionary inflation to transfer resources away from private agents' debt. However, another channel is possible. Revenue might be correlated with some other factors such as a higher preference for government spending, which would increase incentives to use inflation for transfer of resources. This channel could explain our results.

The probability of deviating is negatively related to the target level and significant at the 5% level in all settings. Our interpretation is that some countries might have inflation targets that are too low, making it more likely to exceed the target more often. Those counties could improve their ability to keep inflation on target by adopting higher targets. The results remain little changed when including shocks to exchange rates, the output gap, or a time dummy. Changes in the real effective exchange rate seem to be an important factor in causing policymakers to deviate. The output gap is not significant.

Appendix C Online Appendix: Productivity Cost of Inflation

In this section we present an alternative model in which inflation, instead of affecting utility directly, reduces the endowment of agents as inflation becomes higher. The rest of the model is identical, and results are nearly identical.

The policymaker maximizes an intertemporal utility with the same assumptions as the main model:

$$\max_{\pi_t, D_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t) \tag{38}$$

but is now subject to the following budget constraint:

$$g_t + (1 + r_t)D_t \leq D_{t+1} + \alpha_t \tau e \ , \tag{39}$$

The fixed endowment is subject to a penalty α_t that depends on the policymaker's choice of inflation. The penalty function α is divided into two components, α^p and α^c. The first component, α^p, depends on the inflation level and reflects the productivity cost of the inflation level on output. We assume a productivity cost of inflation function of the form of

$$\alpha^p(\pi) = (1 - \sigma) + \sigma e^{-\lambda \pi^2} \ ,$$

where $1 - \sigma$ is the lower limit on the inflation cost and λ is a fixed parameter. In this setup, $\alpha(0) = 1$, so the optimal inflation level considering only the productivity cost of inflation is zero. The second component is analogous to the violation cost in the main model:

$$\alpha_t^c = \begin{cases} 0 & \text{if } \pi_t = \pi^a, \alpha_{t-1}^c = 0 \\ -\epsilon & \text{if } \pi_t \neq \pi^a, \alpha_{t-1}^c = 0 \\ \alpha_{t-1} & \text{otherwise} \end{cases}$$

We define $\alpha^a = \alpha^p(\pi^a)$, the productivity cost of committing to the inflation target. The optimal discretionary inflation problem can be rewritten as

$$\max_{\pi,D} u(c_T, g_T) + \frac{\beta}{1-\beta} u(c,g)$$

subject to

$$\begin{aligned} g_T &= \alpha(\pi)\tau e - (1+r_T^D)D_T + D \\ g &= \alpha(\pi)\tau e - D\left(\frac{1}{\beta} - 1\right) \\ c_T &= \alpha(\pi)e - g_T \\ c &= \alpha(\pi)e - g \\ 1 + r_T^D &= \frac{1}{\beta}\frac{1}{q_T^e}\frac{1}{1+\pi}. \end{aligned} \tag{40}$$

The solution for the optimal deviation D is identical to the one given by equation 8, but the first-order condition for the discretionary inflation choice changes to

$$(\rho - (1-\rho)v'(g))\frac{\partial(1+r_T^D)}{\partial \pi} D_T + (\rho(1-\tau) + (1-\rho)v'(g)\tau)\,\alpha' e = 0 \tag{41}$$

which has the same intuition as before, but now raising the inflation level affects utility by reducing the available consumption in the steady-state, both for the private and the government consumption goods.

C.1 Calibration and Results

As before, we consider that $v(g) = log(g)$ and calibrate the parameters of the model following the same reasoning of the main model, but we now have an additional parameter σ to calibrate to match the 70% debt-to-GDP crisis zone observed in the Brazilian 2002 inflation crisis:

Insert Table C.I about here.

The main results are presented below. The policy function is nearly unchanged, as the calibration ensures that the crisis zone starts at around 70% of GDP:

Insert Figure C.I about here.

Expected inflation is lower for the model with productivity penalty, which imply a lower benefit of reducing debt rapidly and as a result the policymaker takes longer to exit the FFZ for moderate debt levels.

Insert Table C.II about here.

As before, the deviation from target is decreasing on the target level, and the inflation target serves the same coordinating role of expectations as the model with inflation disutility, raising the FFZ floor as the target is increased. Given the lower discretionary inflation, the optimal inflation target is slightly lower than the previous model.

Insert Figure C.II about here.

Insert Figure C.III about here.

Insert Table C.III about here.

Appendix D Online Appendix: Solution to Discretionary Inflation

Proposition 7 *Let the utility function $u(c, g)$ and the penalty function $\psi(\pi)$ be such that they satisfy the already stated assumptions. If the universe of possible inflation choices is defined on the compact set $[0, \overline{\pi}]$ where $\overline{\pi} > 0$ is some upper limit, then there exists a discretionary inflation level π^D such that π^D is optimal given private agents' inflation expectations π^e and vice versa.*

Proof: To prove that there exists a discretionary inflation level π^D such that π^D is optimal given π^e, and vice versa, we will use Brouwer's fixed point theorem. Since we are only interested in the universe of limited inflation, we state that $\pi^D \in [0, \overline{\pi}]$ where $\overline{\pi} > 0$ is an upper limit for the possible inflation levels. Let $\pi : [0, \overline{\pi}] \to [0, \overline{\pi}]$ be the function mapping private agents' expectations into the policymaker's inflation choice as defined by the discretionary inflation problem in Equation 7.

Let us now define the auxiliary function $\tilde{\pi}(\pi^D) := \pi(f\pi^D + (1 - f)\pi^a) = \pi(\pi^e)$. Since $\tilde{\pi} : [0, \overline{\pi}] \to [0, \overline{\pi}]$ maps a compact interval on $\mathbb{R}$ into itself, we only need to prove that it is continuous to use Brouwer's theorem for the existence of a fixed point.

First, by assumption, we know that the penalty function $\psi : [0, \overline{\pi}] \to \mathbb{R}_+$ mapping discretionary inflation into total factor productivity is continuous, assuming that the policymaker already chose to deviate from the target. Hence, the consumption choice will also be continuous. The same holds for government spending.

Second, the utility function $u : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ mapping government spending and private consumption into a utility scale is also continuous by assumption.

Combining the mapping of discretionary inflation $[0, \overline{\pi}]$ into consumption and spending $\mathbb{R}_+ \times \mathbb{R}_+$ and the mapping of consumption and spending $\mathbb{R}_+ \times \mathbb{R}_+$ into a utility scale $\mathbb{R}$, it is clear that the mapping of discretionary inflation $[0, \overline{\pi}]$ into a utility scale $\mathbb{R}$ will also be continuous. Finally, given that the argmax operator, mapping $[0, \overline{\pi}]$ into $[0, \overline{\pi}]$, maintains those properties, we have that $\tilde{\pi} : [0, \overline{\pi}] \to [0, \overline{\pi}]$ is continuous, which is what we sought to demonstrate.

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A Tables

Parameter	Value	Meaning	Calibration
β	.915	Discount factor	Ex-post 1996-2019 real interest rate
τ	35%	Tax rate	General gov. revenue in % of GDP
π^a	3.5%	Inflation target	2002 BCB target
f	20%	Crisis prob.	EMBI+ Brazil on 10/2002
e	1.5	Endowment	Expansive gov.
ρ	.50	Pref. for consumpt.	Neutral value
ϵ	.004	Fixed cost	Brazilian 2002 crisis
κ	0.35	Welfare cost	Campos and Cysne (2018) estimation for 10% inflation cost

Table I: Parameters of the Baseline Model

Initial Debt Level	Debt Zone	Expected Inflation	Years to Exit the FFZ
60%	Credibility	3.5%	-
75%	Fiscal Fragility	4.6%	1 year
85%	Fiscal Fragility	4.9%	3 years
100%	Fiscal Dominance	12.8%	-

Table II: Expected Inflation Rates and the Time at Which the FFZ is Exited in the Calibrated Model for the Brazilian Case

Initial Debt Level	Optimal Inflation Target
65%	2.4%
75%	4.5%
85%	6.4%

Table III: Optimal Inflation Target for Initial Debt Levels in the Calibrated 2002 Brazilian Case

Year	Date When Set	Target	Bounds
2002	28/6/2000	3.50	2.0
2003	28/6/2001	3.25	2.0
	27/6/2002	4.00	2.5
	21/1/2003	8.50	
2004	27/6/2002	3.75	2.5
	21/1/2003	5.50	
	25/6/2003	5.50	2.5
2005	25/6/2003	4.50	2.5

Table IV: Brazil – Official Inflation Targets

	Debt/GDP	Revenue/GDP	Expected CPI	CPI target
Average	45.2	32.9	3.9	3.2
Min	13.4	16.4	1.5	1.5
Max	80.8	56.1	15.4	8.2

Table B.I: Data Description

	I	II	III	IV	V
Revenue	0.171^{**}	0.098	0.063	0.125^{*}	0.087
	(0.076)	(0.076)	(0.078)	(0.070)	(0.072)
Debt	0.069^{*}	0.074^{**}	0.073^{**}	0.062^{**}	0.058^{*}
	(0.035)	(0.034)	(0.034)	(0.031)	(0.032)
Debt * Revenue/100	-0.194^{*}	-0.168^{*}	-0.149	-0.163^{*}	-0.136
	(0.099)	(0.096)	(0.096)	(0.088)	(0.089)
Target	-0.403^{***}	-0.458^{***}	-0.441^{***}	-0.360^{***}	-0.342^{***}
	(0.063)	(0.062)	(0.062)	(0.059)	(0.058)
GDP Gap			0.363^{***}		0.342^{***}
			(0.102)		(0.095)
Reer YoY				-13.956^{***}	-13.645^{***}
				(1.648)	(1.653)
FE	Country	Country & Time	Country & Time	Country & Time	Country & Time
R^2	0.290	0.408	0.433	0.515	0.537
Num. obs.	382	382	374	372	364

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table B.II: Results – Deviations from the Inflation Target

	I	II	III	IV	V
Revenue	0.145	0.108	0.084	0.115	0.082
	(0.091)	(0.105)	(0.109)	(0.110)	(0.113)
Debt	0.034	0.055	0.053	0.050	0.042
	(0.044)	(0.052)	(0.053)	(0.053)	(0.054)
Debt*Revenue/100	-0.114	-0.107	-0.088	-0.121	-0.085
	(0.125)	(0.149)	(0.151)	(0.151)	(0.154)
Inflation Target	-0.624^{**}	-1.242^{***}	-1.207^{***}	-0.990^{**}	-0.936^{**}
	(0.263)	(0.376)	(0.376)	(0.390)	(0.386)
GDP Gap			0.206		0.218
			(0.158)		(0.167)
Reer YoY				-10.493^{***}	-10.262^{***}
				(3.062)	(3.101)
Num. obs.	377	377	369	368	360
Log Likelihood	−178.526	−151.367	−149.281	−139.619	−137.954

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table B.III: Results – Probability of Overshooting the Target

Parameter	Value	Meaning	Calibration
β	.914	Discount factor	Ex-post 1996-2019 real interest rate
τ	35%	Tax rate	General gov. revenue in % of GDP
π^a	3.5%	Inflation target	2002 BCB target
f	20%	Crisis prob.	EMBI+ Brazil on 10/2002
e	1.5	Endowment	Expansive gov.
ρ	.50	Pref. for consumpt.	Neutral value
σ	20%	Limit to TFP cost	Brazilian 2002 crisis
ϵ	.002	Fixed cost	Brazilian 2002 crisis
λ	1.77	Welfare cost	Campos and Cysne (2018) estimation for 10% inflation cost

Table C.I: Parameters of the Baseline Model

Initial Debt Level	Debt Zone	Expected Inflation	Years to Exit the FFZ
60%	Credibility	3.5%	-
75%	Fiscal Fragility	4.5%	1 year
85%	Fiscal Fragility	4.7%	4 years
100%	Fiscal Dominance	11.5%	-

Table C.II: Expected Inflation Rates and the Time at Which the FFZ is Exited in the Calibrated Model for the Brazilian Case

Initial Debt Level	Optimal Inflation Target
60%	2%
70%	4%
80%	5.5%

Table C.III: Optimal Inflation Target for Each Debt Level in the Calibrated 2002 Brazilian Case

B Figures

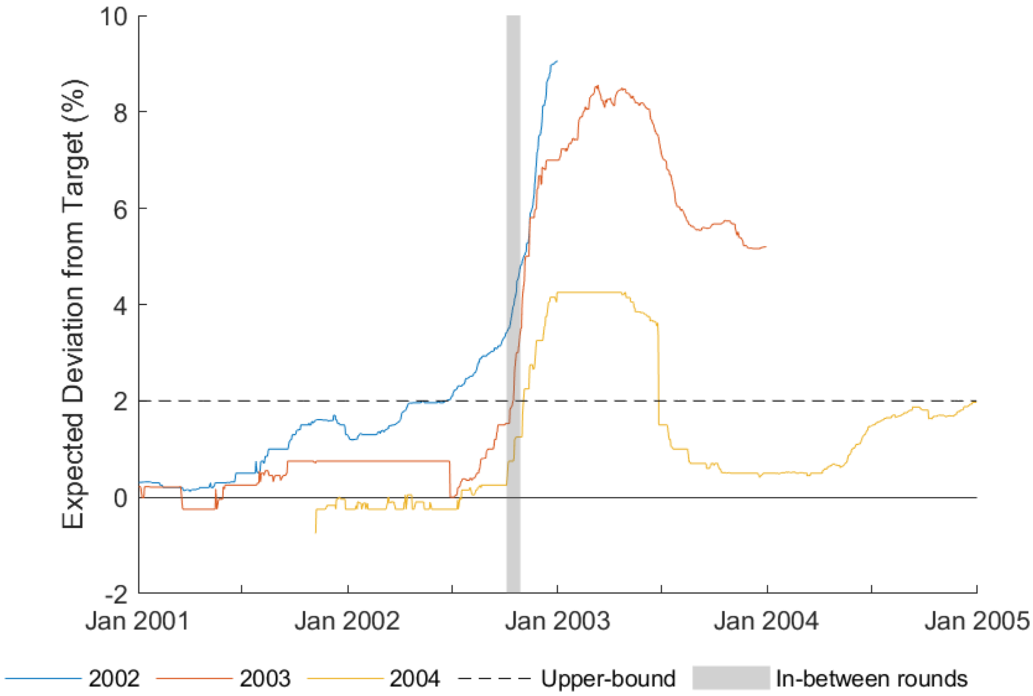

Figure I: Expectation Crisis in Brazil

This figure shows the inflation expectation crisis that happened in Brazil in 2002. On the y-axis, we plot the expected inflation for the end of the year minus the inflation target for that year. Expected inflation is the mean expected inflation by professional forecasters, collected by the Central Bank of Brazil and available at The Focus – Market Readout. On the x-axis, we plot the date when expected inflation was formed. Until October 2002, expected inflation was within the inflation target bands. However, between rounds of the Presidential election – shaded grey region – inflation expectations exceeded the target's upper bounds at all horizons relevant to the central bank (current year, 1-year ahead, and 2-years ahead).

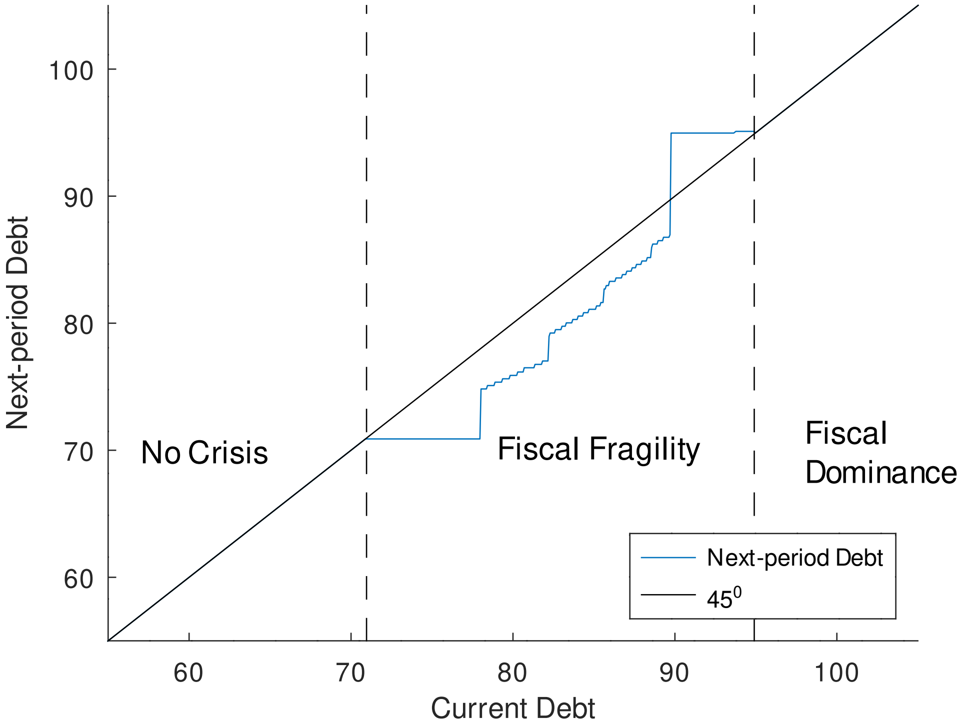

Figure II: Debt Policy Function

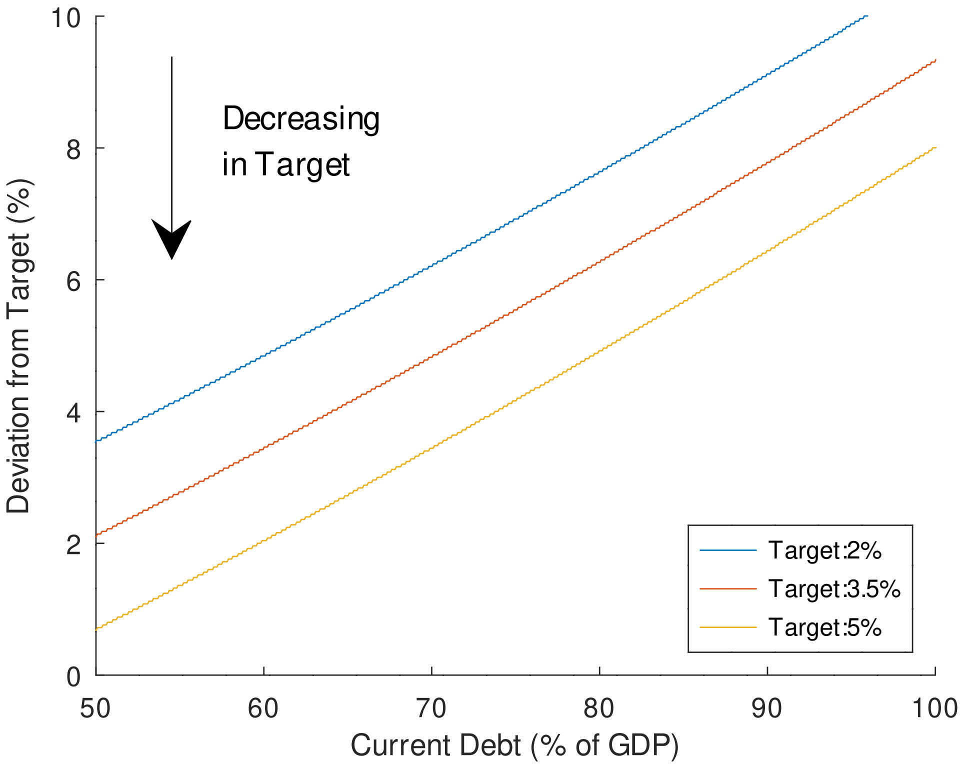

Figure III: Sensitivity: Deviations to Inflation Target

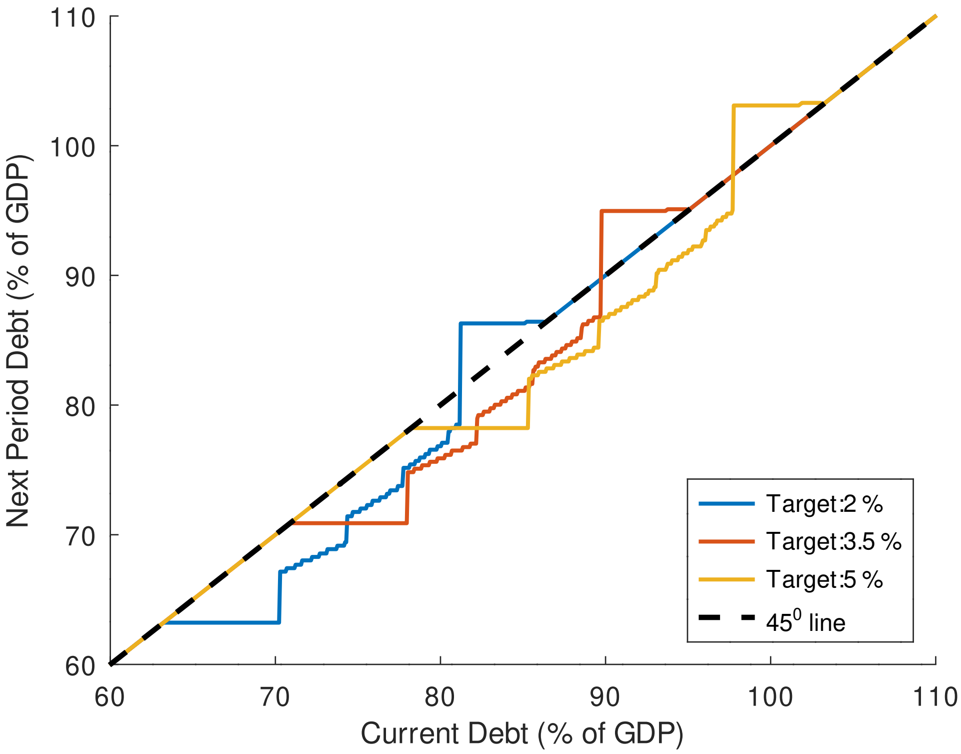

Figure IV: Sensitivity: Inflation Target and the Fiscal Fragility Zone

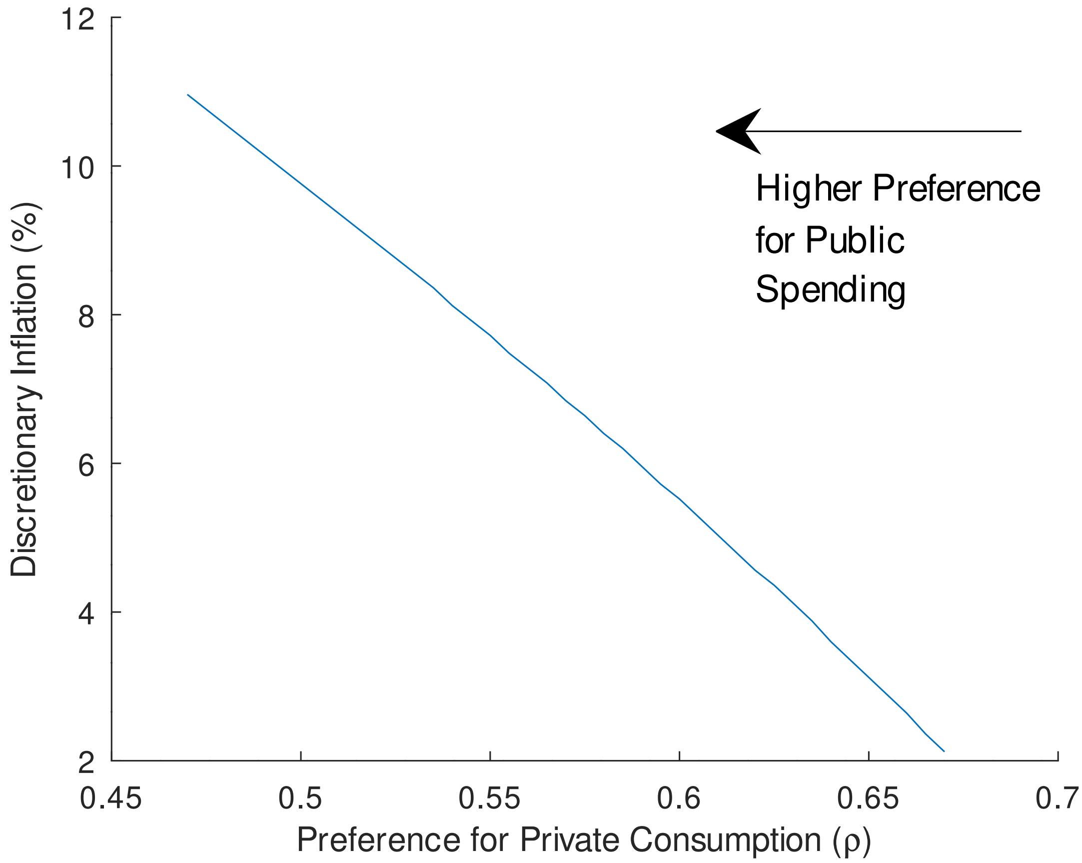

Figure V: Sensitivity: Preference for Public Spending when $D = 80\%$

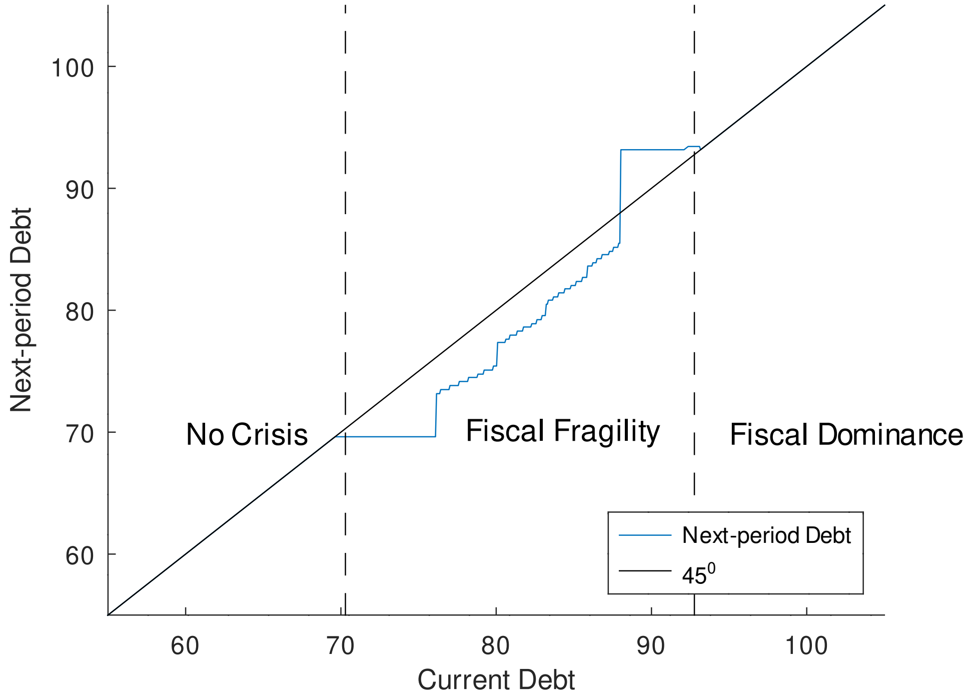

Figure C.I: Debt Policy Function

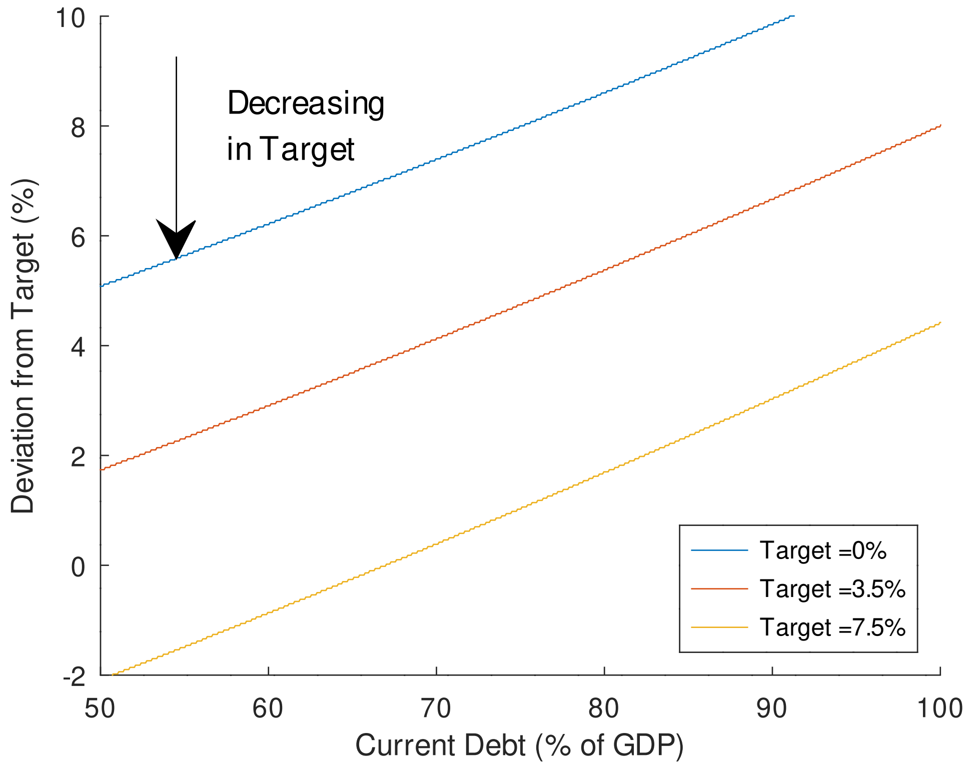

Figure C.II: Sensitivity: Deviations to Inflation Target

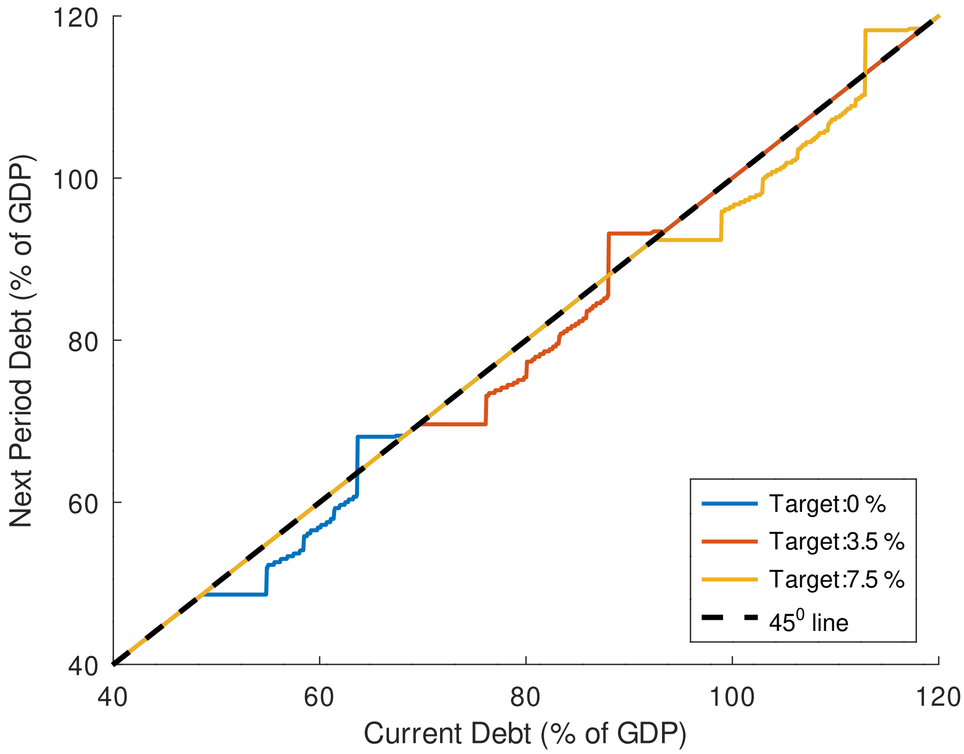

Figure C.III: Sensitivity: Inflation Target and the Fiscal Fragility Zone